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Foundations of quantum mechanics: The Langevin equations for QM

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ABSTRACT

Stochastic derivations of the Schrödinger equation are always developed on very general and abstract grounds. Thus, one is never enlightened which specific stochastic process corresponds to some particular quantum mechanical system, that is, given the physical system—expressed by the potential function, which fluctuation structure one should impose on a Langevin equation in order to arrive at results identical to those comming from the solutions of the Schrödinger equation. We show, from first principles, how to write the Langevin stochastic equations for any particular quantum system. We also show the relation between these Langevin equations and those proposed by Bohm in 1952. We present numerical simulations of the Langevin equations for some quantum mechanical problems and compare them with the usual analytic solutions to show the adequacy of our approach. The model also allows us to address important topics on the interpretation of quantum mechanics.

0. Introduction

In the late 1960s it had appeared in the literature a great number of attempts to derive the quantum mechanical formalism using notions coming from the field of stochastic processes [1]. All these derivations aimed at showing that the overall conceptual and formal frameworks of quantum mechanics could be more easily understood if assessed using stochastic constructs. In fact, all these stochastic approaches have two main common features: (a) they never deviate from usual classical concepts; although always considering notions such as 'fluctuations', 'noises', among others, they are based upon dynamic differential equations (stochastic ones, surely—e.g. Langevin equations) and their constructs

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are always objective. In fact, (b) these approaches are always based upon *corpuscular* models and, within them, undulatory effects must be considered as the outcome of the stochastic behavior (of the particles). Because of this second characteristic, notions as 'observer', 'wave-particle duality', 'reduction of the wave-packet', etc. are not usually found within such approaches.

It is interesting, at this point, to stress that one should discern between classical physics and classical mechanics. We are not saying that stochastic approaches are the same as classical mechanics, since they clearly deviates from Newtonian approaches by considering fluctuations as driving forces. For a theory to be considered "classical" it is only necessary for it to use only *the concepts* in the same way they are used within the classical framework (such as trajectories, observers as merely coordinate systems, etc.) and do not use concepts outside it (such as observer as something beyond a coordinate system, duality, etc.). That is why we have classical electrodynamics, classical statistical mechanics, classical theory of relativity. Each one of these fields disclosed a number of new results, not envisaged by Newtonian Mechanics. They are, nevertheless, classical. Randomness is a mere statistical concept, found in economy, stock market, etc., that cannot change the approach from being classical (except if this randomness implies the need of new concepts outside the classical framework).

Of course, if a stochastic approach mathematically *derives* the Schrödinger equation from considerations about randomness, this means that *there must be some way* by which the ubiquitous appearance of undulatory phenomena in the quantum framework is reduced to corpuscular behavior. However, because of the highly abstract nature of such mathematical derivations, one is never enlightened about how this is actually the case *for particular physical problems*, that is, which specific dynamic-stochastic equation (for particles) furnishes, for some specific physical system, its overall undulatory behavior. Moreover, approaches such as that of de La Peña [2] hardly classify as true stochastic approaches, since they never deal with truly random forces, but with their time averages.

This paper wishes to fill this gap presenting dynamic-stochastic equations in terms of Langevin equations of a particular nature that reproduce all the results obtained from the Schrödinger equation for any quantum mechanical system. The knowledge of such equations would allow us to run simulations of actual physical systems to *show* how the undulatory behavior (e.g. interference) can have its origins in the random movement of particles. If one can show that the undulatory *behavior* of quantum mechanics comes from a corpuscular *nature*, the concept of duality, in its ontological formulation, becomes completely avoidable.

The way by which we construct the adequate Langevin equations depends on some previous results obtained by one of the authors on the foundations of quantum mechanics [3–5] which are briefly presented in the first section. In fact, it can be shown [3] (see the next section) that the Schrödinger equation can be derived from the classical Liouville equation using the characteristic function

$$Z\left(q,\delta q,t\right)=\int \exp\left(\frac{ip\delta q}{\hbar}\right)F\left(q,p,t\right)dp \tag{1}$$

when $Z\left(q,\delta q,t\right)$ is expanded up to second order in δq. This first derivation was shown to be mathematically equivalent to another one in which the notion of entropy replaced the notion of characteristic function [4]. The importance of doing this second derivation comes from the fact that the concept of entropy is more akin to the notion of fluctuation. These two derivations were also shown [5] to be mathematically equivalent to the stochastic derivation of de La Peña [2], and the notion of entropy was linked to the notion of fluctuation by the fluctuation–dissipation theorem. Later on, it was shown [6] that all these approaches could be justified based on the random character of the quantum phenomena and the requirements of the Central Limit Theorem. In [6] it was shown how the proposed approach dramatically reduces the number of ontological entities and explanatory tools needed to interpret the behavior of quantum systems—and how some quantum oddities are thus removed. Some of these developments, however, were presented in the same level of abstractness as those already derived by physicists acquainted with stochastic approaches.

Thus, despite all these achievements, there still remains the above mentioned problem of showing which dynamical-stochastic system would correctly describe the overall behavior of usual and specific quantum systems and how wavelike behavior may *emerge* from an underlying fluctuating corpuscular physical system. As we show in what follows, the previously mentioned achievements, when put together, can furnish the solution about what should be the nature of the Langevin equations to