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Observation of a non-adiabatic geometric phase for elastic waves

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We report the experimental observation of a geometric phase for elastic waves in a waveguide with helical shape. The setup reproduces the experiment by Tomita and Chiao [A. Tomita, R.Y. Chiao, Phys. Rev. Lett. 57 (1986) 937–940, 2471] that showed first evidence of a Berry phase, a geometric phase for adiabatic time evolution, in optics. Experimental evidence of a non-adiabatic geometric phase has been reported in quantum mechanics. We have performed an experiment to observe the polarization transport of classical elastic waves. In a waveguide, these waves are polarized and dispersive. Whereas the wavelength is of the same order of magnitude as the helix's radius, no frequency dependent correction is necessary to account for the theoretical prediction. This shows that in this regime, the geometric phase results directly from geometry and not from a correction to an adiabatic phase.

1. Introduction

Polarization is a feature shared by several kinds of waves: light and elastic waves, for instance, have two transverse polarization modes [1]. The polarization degrees of freedom are constrained to lie in the plane orthogonal to the direction of propagation. This constraint is responsible, in optics, for the existence of a geometric phase. Geometric phases of different kinds have been discovered since the Berry phase [2–8].

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The geometric phase of light was first experimentally observed by Tomita and Chiao [9] using optical fibers. It was later suggested that a geometric phase should exist for any polarized waves [10]. This paper discusses the case of elastic waves when the adiabatic conditions are not fulfilled, a situation which cannot be reached in optics. For polarized waves in a waveguide, the geometric phase differs from zero only if the shape of the waveguide is three-dimensional and is defined even if the time evolution is not cyclic [11].

The fundamental origin of geometric phases lies in the geometric description of the phase space. The existence of a geometric phase is related to the curvature, either local or global, of the phase space. In a first attempt to classify geometric phases, Zwanziger et al. distinguish adiabatic geometric phases [6], the main example of which is a spin in a magnetic field [2]. The geometric phase for the spin is defined if the system evolves adiabatically, such that transitions between spin states are negligible. The direction of the magnetic field must therefore evolve at a rate $1/T$ much smaller than the oscillation frequency between spin eigenstates.

Consider the case of waves propagating in a curved waveguide. The role of the magnetic field's direction is played by the direction of propagation and the phase between the spin eigenstates is the orientation of the linear polarization of the waves. The adiabatic approximation imposes that the evolution rate $1/T$ is much smaller than the wave frequency. In the first experimental evidence for the adiabatic geometric phase, performed in optics by Tomita and Chiao [9], the frequency of light ν was indeed several orders of magnitude larger than $1/T$. Photon spin flip is negligible, therefore the adiabaticity conditions are fulfilled. In Foucault's pendulum [12], a renowned case of classical geometric phase, these conditions are fulfilled as well.

Some geometric phases do not require adiabaticity, such as the Pancharatnam phase [13] and the Aharonov–Anandan quantum phase [4] or certain canonical classical angles [5,14]. Geometric phases have been observed in many fields of science and called different names. In classical mechanics, the geometric phase for adiabatic invariants is often referred to as the *Hannay angle* [3,15]; in knot theory and DNA physics, the name *writhe* is mostly used [16,17].

We consider from now on elastic waves, whose polarization state can be represented as a combination of two linearly polarized states. These are classical waves, quantum transition between polarization eigenstates is not possible. The adiabaticity condition $\nu \gg 1/T$ should therefore not be required to observe a geometric phase. In our experiment, indeed, the frequency and the evolution rate have the same order of magnitude. In this paper, we briefly introduce the geometric phase in a purely geometric picture and compute its value along a helix. We present an experiment designed to measure the geometric phase of elastic waves in a helical waveguide with the condition $\nu \simeq 1/T$. Without loss of generality, we only consider linear polarization.

Although the experiment we investigate has many common points with previous studies, some distinctions must be pointed out. Contrary to light polarization, the phase cannot be interpreted as a quantum phase difference between two eigenstates. There is no established classification of geometric phase but as the system we study is purely classical and neither cyclic nor adiabatic the observed geometric phase cannot be rigorously identified as a Berry, Pancharatnam, Aharonov–Anandan or Wilczek–Zee phase, for instance. The classical geometric angles [5,14] follow from the action-angle representation of the system, which is valid for the motion of a material point of the waveguide, but does not rigorously apply to the elastic wave transport.

2. The geometric phase for polarized waves

A wave travelling along a straight path keeps a constant polarization along the trajectory; if the direction of propagation is not constant, as the polarization is ascribed to remain in the orthogonal plane, it evolves along the path. The path-dependent transformation transporting the polarization must be, for physical reasons, linear, reversible and continuous. There is only one transformation satisfying these requirements: *parallel transport* [10]. Along the path followed by the wave, the direction of propagation is represented as a point on the unit sphere, the polarization is represented in the tangent plane to the sphere and transported in the sphere's tangent bundle, see Fig. 1.

Let us consider a geodesic on the unit sphere, *i.e.* an arc of a great circle. If the polarization is collinear or orthogonal to the great circle, it remains so along the geodesic to preserve symmetry. Any