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Hyperbolic Bloch equations: Atom-cluster kinetics of an interacting Bose gas

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HIGHLIGHTS

- Atom clusters applied to efficiently describe an interacting Bose gas.
- Implicit-notation formalism delivers quantum kinetics of *all* atom clusters.
- Common cluster-expansion approach is found for BEC and semiconductors.
- Hyperbolic Bloch equations introduced to analyze fast-switch experiments.

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ABSTRACT

Experiments with ultracold Bose gases can already produce so strong atom–atom interactions that one can observe intriguing many-body dynamics between the Bose–Einstein condensate (BEC) and the non-condensed atoms. This dynamics is thoroughly analyzed with the cluster-expansion approach to uniquely identify atom-cluster dynamics within the many-body system. These clusters assign those atoms that are genuinely connected with one another. The excitation picture is applied to express the many-body state in terms of correlated atom clusters among the non-condensed atoms alone. Implicit notation formalism is developed to *explicitly* derive the quantum kinetics of *all* atom clusters. The clusters are shown to build up sequentially, from smaller to larger ones, which is utilized to nonperturbatively describe the interacting BEC with as few clusters as possible. This yields the hyperbolic Bloch equations (HBEs) that not only generalize the Hartree–Fock Bogoliubov approach but are also analogous to the semiconductor Bloch equations (SBEs). This connection is utilized to apply sophisticated many-body techniques of semiconductor quantum optics to BEC investigations. Here, the HBEs are implemented to determine how a strongly interacting Bose gas reacts to a fast switching from weak to strong interactions, often referred to as unitarity. The

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computations for ^{85}Rb demonstrate that molecular states (dimers) depend on atom density, and that the many-body interactions create coherent transients on a 100 μs time scale converting BEC into non-condensed atoms via quantum depletion.

1. Introduction

Ultrafast spectroscopy [1–10] is originally a concept of optics that is based on exciting and probing matter with laser pulses. When such pulses switch the light–matter interaction on and off on a time scale much faster than the fundamental scattering processes, one can monitor [11,12] and even control quantum processes [13–16] on their natural time scales. In practice, one typically uses picosecond to attosecond laser pulses depending on how fast the studied phenomena are. By monitoring the excitation dynamics that follows, one can directly investigate, e.g., the progress of chemical reactions [17,18]. Ideally, this method can reveal how one can control the coherent dynamics, governed by the Schrödinger equation, in order to influence the late-time phenomena despite the onset of inevitable relaxation and thermalization. This approach is particularly interesting in semiconductor optics [19–28,10,29] and quantum optics [8,30,31] because semiconductor excitations can exhibit a great variety of quantum processes that proceed on different time scales. As hallmarks of genuine quantum kinetics, perturbative as well as thermodynamic approaches fail [32–35], coherent control [13–16] becomes possible, and quantum beats [11,12] or even the quantum fluctuations of light [36–38] can influence the generation of new states and configurations.

Ultrafast control of many-body systems offers the possibility of exploring how elementary quantum events translate into macroscopic phenomena, which is probably the most challenging problem of contemporary physics. Therefore, it is most fruitful to engage an approach I call manybodypedia, a synergistic learning process where different aspects of many-body quantum physics are systematically analyzed from a point of view of seemingly different systems. We are very much experiencing this progress already: a lattice of ultracold atoms is applied to simulate [39–42] solid-state models, atomic Bose–Einstein condensation is being pursued [43–45] with semiconductors, Efimov physics [46] of nuclei is studied in interacting Fermi [47–49] as well as Bose [50–53,49,54] gas of atoms, to mention only a few examples.

In this context, atomic Bose–Einstein condensates (BECs) [55–57] are perhaps the cleanest example manifesting macroscopic quantum phenomena. Since such a Bose gas contains interacting atoms, it clearly is interesting to study what kind of quantum kinetics results from the atom–atom interactions. In the spirit of manybodypedia, one can use the ideas of ultrafast spectroscopy because one can change the strength of atom–atom interactions faster than relaxation by sweeping the system through a Feshbach resonance [58–65] with an external magnetic field. In particular, recent experiments [66–68] in interacting Bose gas demonstrate that one can switch the interactions on a time scale much faster than essential scattering rates. This means that one can start exploring analogous nonequilibrium many-body quantum kinetics as that performed in semiconductors using ultrafast spectroscopy [30]. Obviously, the time scales of interacting Bose gas are very different from those encountered in ordinary ultrafast spectroscopy because fast Bose gas processes proceed on a submillisecond time scale due to extremely slow motion of ultracold atoms, in contrast to subpicosecond times for quantum processes in solids.

In this paper, I will develop a general framework to describe many-body quantum kinetics of interacting Bose gas following a fast switching of interactions. The introduced method is based on the cluster-expansion approach presented in Refs. [69,70,25,71,30] investigating ultrafast as well as quantum-optical semiconductor spectroscopy. Cluster-expansion based approaches are also used in quantum chemistry [72,73] in the form of the coupled-clusters approach that was originally developed to explore the many-body physics of nuclei [74,75]. Therefore, it is fair to say that the cluster