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Bell states and entanglement dynamics on two coupled quantum molecules

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ABSTRACT

This work provides a complete description of entanglement properties between electrons inside coupled quantum molecules, nanoestructures which consist of two quantum dots. Each electron can tunnel between the two quantum dots inside the molecule, being also coupled by Coulomb interaction. First, it is shown that Bell states act as a natural basis for the description of this physical system, defining the characteristics of the energy spectrum and the eigenstates. Then, the entanglement properties of the eigenstates are discussed, shedding light on the roles of each physical parameters on experimental setup. Finally, a detailed analysis of the dynamics shows the path to generate states with a high degree of entanglement, as well as physical conditions associated with coherent oscillations between separable and Bell states.

1. Introduction

In the last two decades, the interest on the physical behavior of semiconductor materials has increased due to their potential applications in quantum computing [1,2]. In particular, semiconductor quantum dots have been proved to be an ideal candidate for the codification of quantum information. In these systems, a qubit can be defined using the charge [3] and the spin of the particle confined [4–6], as well as excitonic states [7,8]. An interesting system is a quantum molecule (QM) which is a nanoestructure consisting on two quantum dots separated by a barrier that can be tunneled by charged particles, as electrons [9–12].

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In 2003, Hayashi et al. demonstrated the coherent manipulation of a charge qubit in a QM [13]. The quantum nanoestructure is built by using metal gates for confining charges on a bidimensional electron gas in GaAs/AlGaAs. The same group implemented a quantum device with two QMs coupled electrostatically but isolated by conduction [14]. In this system, the resonant tunneling current through each molecule is influenced by the charge state of the second molecule [15]. More recently, researchers on the Quantum Device Lab (ETH Zurich) and Université of Sherbrooke (Québec, Canada) demonstrated successfully the coupling between the same type of device with a microwave resonator [16]. The experimental data (together with a theoretical treatment) shows that the coupling between a quantum molecule and the resonator follows a Jaynes–Cummings type interaction. In another interesting work, quantum oscillations between three quantum states in a Si/SiGe double quantum dots are controlled by high-speed voltage pulses [17]. Measurements of the transconductance permitted the observation of this oscillations between different coupled quantum states induced by two different pulse profiles. Both works show the potential applications of this kind of heterostructures on quantum information processing, opening the possibility of the definition and control of quantum bits.

From the theoretical point of view, Fujisawa et al. [18] demonstrated that, in the experimental setup mentioned above, it is possible to implement some of the two-qubit gates, including the Bell gate at a very specific choice of physical parameters. Other theoretical works have explored aspects related to quantum correlations and decoherence [19,20], also considering restricted physical conditions. Due the importance of the role of entanglement in quantum computation protocols, a complete analysis of the entanglement properties of this particular system, with potential application of quantum information processing, is crucial.

Here, it is developed a careful exploration of the entanglement properties of electrons in coupled QMs, considering the realistic experimental setup of Ref. [14], treated as a closed system. The results show that Bell states are the key behind the physical behavior of the electrons inside the coupled QMs. Also, it is demonstrated how the careful control of physical parameters as tunneling, electronic energy offsets (detunings), and Coulomb coupling, can be used to create highly entangled states. First, using analytical calculations together with numerical simulations, the characteristics of the energy spectrum and the eigenstates of the Hamiltonian are explored. The results show that the eigenstates correspond to Bell states under specific physical conditions. Second, the effect of physical parameters is mapped, focusing on the generation of highly entangled states. The "beats" on the dynamics of entanglement are verified and explained as a competition between two different frequencies associated with the physical couplings. Physical requirements to obtain coherent oscillations between separable states and a specific Bell state are discussed. All results are obtained considering realistic values of physical parameters.

This paper is structured as follows. Section 2 contains the description of the theoretical model together with the definition of the measurement of entanglement, concurrence, used in this work. Section 3 is devoted to explore the characteristics of the energy spectrum and the eigenstates from the point of view of entanglement properties. Section 4 is reserved to the discussion of dynamics, focusing in the obtention of Bell states using temporal evolution, considering the effects of tunneling and detuning between electronic states inside each QM. The work is summarized in Section 5.

2. Model

The analysis performed here is based on an actual experimental setup with two coupled QMs [14,13], each with two quantum dots separated by a potential barrier which can be tunneled by electrons [9–12]. The quantum dots for each QM are arranged horizontally, being coupled to a source and a drain of electrons, used to charge and discharge the QM. Coulomb blockade and the physical design of the nanoestrutures guarantee that there is only one extra electron and, effectively, one electronic level for each quantum dot. Each QM can be treated theoretically as a two-level system: the electron can occupy the left (right) dot $|L\rangle$ ($|R\rangle$). Even if the two QMs are designed in order to inhibit intra-molecules tunneling, both subsystems are still coupled because of the Coulomb interaction between the electrons on each molecule. From the point of view of quantum information, this corresponds to the implementation of two qubits, one on each QM, which are coupled in such a way that the physical system is bipartite.