Annals of Physics 356 (2015) 269–298

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Particle diagrams and statistics of many-body random potentials

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ARTICLE INFO

Article history:
Received 6 December 2014
Accepted 7 March 2015
Available online 14 March 2015

Keywords:
Embedded ensembles
Many-body problems
Random matrix theory
Quantum (hyper)graphs

ABSTRACT

We present a method using Feynman-like diagrams to calculate the statistical properties of random many-body potentials. This method provides a promising alternative to existing techniques typically applied to this class of problems, such as the method of supersymmetry and the eigenvector expansion technique pioneered in Benet et al. (2001). We use it here to calculate the fourth, sixth and eighth moments of the average level density for systems with m bosons or fermions that interact through a random k-body Hermitian potential ($k \leq m$); the ensemble of such potentials with a Gaussian weight is known as the *embedded Gaussian Unitary Ensemble* (eGUE) (Mon and French, 1975). Our results apply in the limit where the number l of available single-particle states is taken to infinity. A key advantage of the method is that it provides an efficient way to identify only those expressions which will stay relevant in this limit. It also provides a general argument for why these terms have to be the same for bosons and fermions. The moments are obtained as sums over ratios of binomial expressions, with a transition from moments associated to a semi-circular level density for $m < 2k$ to Gaussian moments in the dilute limit $k \ll m \ll l$. Regarding the form of this transition, we see that as m is increased, more and more diagrams become relevant, with new contributions starting from each of the points $m = 2k, 3k, \ldots, nk$ for the $2n$th moment.

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http://dx.doi.org/10.1016/j.aop.2015.03.009

1. Introduction

Random Matrix Theory (RMT) is the study of random matrices with various symmetry conditions imposed on the matrix entries. With scant warning this young theory has permeated nearly every area of modern physics and even number theory [1–3]. In quantum physics, random matrices can be used to model the behaviour of Hamiltonians or scattering matrices, and many statistical properties of chaotic quantum systems have been found to agree with the appropriate predictions from RMT [3,4]. In recent decades attempts have been made to further refine what has become the canonical theory, by considering symmetries which allow the matrix representations of quantum potentials to impose k-body interactions between the particles in a system containing m particles ($k \leq m$). Here the main new feature is that such an interaction, when applied to one of the states with m particles, will annihilate k particles and create k particles in (possibly) different single-particle states. This means that matrix elements between many-particle states that differ by more than k occupied single-particle states will necessarily be zero. Canonical RMT, containing no such restrictions, can be associated with the case $k = m$. The case $k = 1$ describes random single-particle potentials and hopping terms. Although the most common interactions have $k = 2$ it remains of great interest to determine the statistics of such interactions for the whole physically relevant domain $k \leq m$.

The appropriate generalisation of canonical RMT involves *embedding* the k-body potential into the m-particle state space creating what has become known as the embedded ensembles. The embedded ensembles, first introduced by Mon and French [5] in 1975, gave physicists a powerful framework for studying many-body interactions using random matrix theory. (See [6,7] for reviews, and [8,9] for the related two-body random ensemble.) In particular, the embedded Gaussian Unitary Ensemble of random matrices (eGUE) represents the Hamiltonian of non time-reversal invariant quantum systems of m particles interacting under the force of a k-body potential, so called because the potential is a sum of interaction terms between k-tuples of particles. In addition, many-body Hamiltonians of a similar form are used independently to study the statistics of quantum spin chains, spin glasses and (hyper)graphs [10–12] and recent developments point to a convergence of some statistical properties between these models [12–14].

In one of the main contributions to this area Benet, Rupp and Weidenmüller [15] showed how a process of eigenvector expansions could be used to calculate certain statistical properties of k-body potentials, in particular the fourth moment of the average level density. Though a great advance, the eigenvector expansion method is complex to implement, and it remains unclear if it can practically be used to calculate moments higher than the fourth. The method of supersymmetry, also used in [15] to investigate the fourth moment, is accompanied by technical difficulties in the loop expansion, and does not allow one to access the regime $m \geq 2k$. A further technique used to treat embedded ensembles is the trace propagation method [7]. Using a new method however, which utilises Feynman-like diagrams to simplify calculations, we will show that it becomes possible to calculate the fourth, sixth and eighth moments for embedded ensembles in a straightforward way. The method, which we will call the method of *particle diagrams*, is designed to probe the order of magnitude of combinatorial expressions prior to calculating them explicitly. We will specifically be interested in the case where, in correspondence with many physical systems, the number of available single-particle states l is taken to infinity. In this limit estimating the order of magnitude provides a sufficient excuse not to calculate certain terms at all, since we can foretell using particle diagrams that they will not survive in this asymptotic regime. Hence by applying the method of particle diagrams one is in effect washing out much of the complexity of the problem, with enough details remaining to yield limiting statistics.

We will present this technique in detail here, significantly extending our previous rapid communication [13]. First, we will introduce the method using the fourth moment as an example. Afterwards we will proceed to the sixth and eighth moments, using a further methodological development that involves studying closed loops on particle diagrams. It will be shown that the (normalised) fourth moment of the eGUE is given by the combinatorial expression

$$\kappa \sim 2 + \frac{\binom{m-k}{k}}{\binom{m}{k}} \tag{1}$$