Annals of Physics 356 (2015) 306–319

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Phase-space moment-equation model of highly relativistic electron-beams in plasma-wakefield accelerators

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ARTICLE INFO

Article history:
Received 27 November 2014
Accepted 6 March 2015
Available online 14 March 2015

MSC:
82D10
35Q83

Keywords:
Plasma physics
Plasma-based acceleration
Vlasov equation
Relativistic electron-beams

ABSTRACT

We formulate a new procedure for modelling the transverse dynamics of relativistic electron beams with significant energy spread when injected into plasma-based accelerators operated in the blow-out regime. Quantities of physical interest, such as the emittance, are furnished directly from solution of phase space moment equations formed from the relativistic Vlasov equation. The moment equations are closed by an Ansatz, and solved analytically for prescribed wakefields. The accuracy of the analytic formulas is established by benchmarking against the results of a semi-analytic/numerical procedure which is described within the scope of this work, and results from a simulation with the 3D quasi-static PIC code HiPACE.

1. Introduction

Plasma-based accelerator technology [1] provides gradients for the acceleration of charged particles in excess of 10 GV/m, which surpasses the fields in conventional accelerators by three orders of magnitude. For this reason, plasma accelerators promise a dramatic miniaturization of electron accelerator technology and accordingly raise hopes for a significant reduction of their cost. The field of plasma acceleration has made remarkable progress in the past decade with the demonstration of mono-energetic spectral features [2–4], the breaking of the GeV-energy barrier [5], controlled

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http://dx.doi.org/10.1016/j.aop.2015.03.004

bunch-injection techniques for improved beam quality [6–9], improved stability [10] and high efficiency [11]. Also, first applications for photon generation from plasma-generated electron beams have been demonstrated [12,13].

Despite this tremendous development, relativistic electron beams from plasma accelerators are not yet achieving the phase-space quality required for more demanding applications such as free-electron lasers and high-energy physics colliders. Further progress in this area critically depends on obtaining a better understanding of, and thus, control over the key processes, such as those influencing transverse beam properties. It is here that theoretical modelling can play a significant role. PIC (particle in cell) codes [14] have hitherto provided the most detailed insight, yet, require significant computational resources. Analytical modelling offers in many cases a complementary approach without the need for extreme computation. The procedures outlined in this paper have been developed to that end, and provide an analytical way of describing the evolution of the transverse phase space of an electron beam propagating in a plasma wave along a longitudinally homogeneous plasma-profile. In addition, a numerical method is proposed, which models the transverse dynamics of an electron beam in a plasma wakefield in inhomogeneous plasma in a computationally efficient manner.

We start with the observation that the quantities of experimental interest may be expressed as phase space averages or "moments" of the electron phase space distribution function. The latter may be found as the solution of the relativistic Vlasov kinetic equation, but we prefer to work instead solely with equations for the moments themselves, which are found by forming appropriate averages of the Vlasov equation. In other words, we seek to find the averages without the need to first find the distribution function. The price of taking this "short cut" (see Fig. 1) is that it produces more unknowns than equations, the familiar problem of closure, and the challenge is then to find an *Ansatz* (or postulate) which closes the moment equations while still producing acceptably accurate solutions. As with fluid modelling in plasma physics [15], an Ansatz built on sound physical reasoning, rather than ad hoc supposition, is an essential ingredient for success.

In this paper we consider a relativistic electron bunch with energy spread which is injected into wakefields, driven by a laser or particle beam in the blowout regime [16]. The field configuration in this regime is featured by focusing fields which are proportional to the radius from the propagation axis of the driving beam [17,18]. Moment equations are built from first principles, and solved rigorously for the case where the driver beam does not evolve and the focusing fields remain constant along the direction of driver propagation. This is a valid assumption if the injected beam is ultra-relativistic, such that the betatron wavelength is long compared to the transition from the vacuum to the plasma, and if the driver beam does not evolve significantly during the time over which the expected quality degradation of the witness beam occurs. Analytic expressions for all physical properties of interest, including the emittance, are thereby obtained. In the case where the field varies along the axis of propagation, a semi-analytic/numerical approach is used to furnish the required expressions. The aim is to obtain a computationally economic method for modelling the transverse beam dynamics. The analytic results are benchmarked against the semi-analytic/numerical procedure and results from the quasi-static three-dimensional (3D) PIC code HiPACE [19].

Other procedures aimed at providing an analytic perspective [20,21] start at the single particle level, and proceed to an ensemble average through a series of steps involving an assumed form of the phase space distribution function. In contrast, we consider average properties from the outset, as furnished by phase space moments of the Vlasov equation [22] without stipulating the shape of the phase space distribution explicitly. It is emphasized that the procedure is completely self-contained and we do not need to solve the Vlasov equation for the distribution function itself. In passing we note a similar treatment by Kumar for low energy, non-relativistic electron "swarms" in gases starting with the classical Boltzmann kinetic equation [23].

The structure of this paper is as follows: In Section 2, we discuss the formalism for the phase space moment method and establish the general form of the moment equation. This is followed by discussions of the solution of the equations, both general aspects and the specific details, including the semi-analytic/numerical procedure and the Ansatz, in Sections 3 and 4 respectively. In Section 5, we compare the analytic results with semi-analytic/numerical values and results obtained from a PIC simulation, thereby establishing the credentials of the moment method and the accuracy of the Ansatz. Section 6 summarizes the results and points the way to further applications.