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On the measurability of quantum correlation functions

Bertúlio de Lima Bernardo [*], Sérgio Azevedo, Alexandre Rosas

Departamento de Física, CCEN, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-900, João Pessoa, PB, Brazil

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ABSTRACT

The concept of correlation function is widely used in classical statistical mechanics to characterize how two or more variables depend on each other. In quantum mechanics, on the other hand, there are observables that cannot be measured at the same time; the so-called incompatible observables. This prospect imposes a limitation on the definition of a quantum analog for the correlation function in terms of a sequence of measurements. Here, based on the notion of sequential weak measurements, we circumvent this limitation by introducing a framework to measure general quantum correlation functions, in principle, independently of the state of the system and the operators involved. To illustrate, we propose an experimental configuration to obtain explicitly the quantum correlation function between two Pauli operators, in which the input state is an arbitrary mixed qubit state encoded on the polarization of photons.

1. Introduction

Correlation functions are used in the study of both deterministic and probabilistic systems to describe the dependence of random dynamical variables at two or more distinct points in space or time. They are found to play a fundamental role in many fields, including statistical mechanics [1], astronomy [2], econophysics [3], as well as in the analysis of chaotic processes [4]. From a classical perspective, if we are given a physical system in which a set of random variables $A_1(\xi_1), A_2(\xi_2), \ldots, A_n(\xi_n)$

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[*] Corresponding author.
E-mail address: bertulio.fisica@gmail.com (B. de Lima Bernardo).

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are to be measured at different points of space (or time) $\xi_1, \xi_2, \ldots, \xi_n$, the correlation function is defined as the ensemble average of the product of the measured variables $\langle A_1(\xi_1)A_2(\xi_2)\ldots A_n(\xi_n)\rangle$. Note that, in principle, we can make any measurement on a classical system without disturbing it. As a consequence, a sequence of measurements can also be performed while still preserving its evolution as though no external interference had occurred. This fact ensures the measurability of correlation functions in the classical realm.

In quantum mechanics, we encounter correlations between commuting observables which frequently appear when describing quantum optical experiments. For example, in estimating the second order correlation function of a light source [5,6], which gives us information about the photon statistics, one is effectively measuring the time average of the number of photons registered on two different detectors at two specific times, as it is the case in the Hanbury Brown–Twiss Interferometer [7]. However, the definition of a correlation function analog, as discussed in the classical sense depicted above, is far from obvious. First, the aforementioned dynamical variables give place to the notion of observables, which are represented by self-adjoint linear operators that do not necessarily commute. Second, the very act of measuring disturbs the quantum system in a generally unpredictable way. These two features lead to the concept of incompatible observables (those whose outcomes cannot be known at the same time). In this form, a direct interpretation of the quantum correlation function in terms of a sequence of measurements appears to be problematic. Indeed, if the outcomes of two or more operators are not simultaneously defined, in principle, we cannot measure the average of the product of these operators. Nevertheless, despite the operational significance has not yet been completely clarified, special forms of quantum correlation functions take place in many fields ranging from quantum thermodynamics [8] and quantum statistical mechanics [9] to quantum field theory [10]. Then, any attempt to step towards an unambiguous understanding of such correlation functions is both important and necessary.

In this work, we introduce a simple protocol to directly measure general quantum correlation functions for any mixed input state $\hat{\rho}$ and, in principle, any set of operators $\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n$. The method is based on the concept of sequential weak measurements, in which the quantum system is only slightly disturbed, and has been shown to obtain simultaneous information about non-commuting observables [11]; something prohibited in the standard (strong) measurement regime. In this context, an alternative approach to measure such correlation was proposed by Anastopoulos [12], in which interference phases between different states must be measured in a similar fashion to the Aharonov–Bohm effect and Berry phase measurements [13]. More recently, a probabilistic "black-box"-like strategy was proposed to measure the simplest case of the quantum correlation function of two operators (two-point correlation function), where an optical implementation of the idea was suggested [14]. However, in the experimental proposal only the real part of the correlation function could be obtained. Here, we use our method to circumvent these limitations by presenting an all-optical experimental setup in which both the real and imaginary parts of a two-point correlation function can be measured, where the input state is a mixed ensemble of qubits encoded on the polarization of photons, and the observables are represented by two Pauli operators.

2. Weak measurements of pure and mixed states

We begin by reviewing the theory of weak measurements which is at the heart of our proposal. Roughly twenty-six years ago, Aharonov, Albert and Vaidman (AAV) devised a method that would enable experimenters to perform measurements on a quantum system without destroying its original state [15]. That idea apparently in conflict with Heisenberg's uncertainty principle, one of the cornerstones of quantum theory, remained little noticed by most physicists until the last decade. Notwithstanding, in what concerns the several applications, weak measurements have now been used, for example, to amplify the measurement of small effects that are perpendicular (or parallel) to the propagation direction of a light beam [15–22], while still providing a promising and attractive avenue for future applications in metrology [23,24]. It has also been used in obtaining the average trajectories of single photons in a two-slit interferometer [25], and in the direct measure of a wavefunction [26]. Moreover, weak measurements have given a different view in addressing some foundational