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Dynamical systems study of chameleon scalar field

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ABSTRACT

The present work is an extensive study of the viable stable solutions of chameleon scalar field models leading to possibilities of an accelerated expansion of the universe. It is found that for various combinations of the chameleon field potential $V(\phi)$ and the coupling $f(\phi)$ of the chameleon field with matter, a stable solution for an accelerated expansion is quite possible. The investigation provides a diagnostics for the stability criteria for all sorts of combinations of $V(\phi)$ and $f(\phi)$.

1. Introduction

Although the observational evidences in favour of the recent accelerated expansion of the universe are mounting everyday, the driver of this acceleration has neither been detected nor does any of the candidates has a clear verdict on theoretical grounds. A cosmological constant is the most favoured one, but it has its share of problems [1–3]. Amongst a host of other possibilities, a scalar field with a potential, leading to an effective negative pressure, popularly called a quintessence field, appears to enjoy the second highest priority [4]. Relativistic theories of gravitation, other than the General Theory of Relativity, are also invoked. For example, an $f(R)$ theory of gravity, where the Ricci scalar R in the Einstein action is replaced by an analytical function $f = f(R)$, does well in explaining the accelerated expansion [5]. Another popular direction is to look for an accelerated model in an already existing nonminimally coupled scalar field theory, particularly Brans–Dicke theory or some of the modifications of the theory [6]. In such theories, the scalar field or some function of the same is

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nonminimally coupled to the gravity sector in the action in the form $f(\phi)R$. Recently another form of a nonminimally coupled scalar field theory, where the scalar field is coupled to the matter sector rather than the geometry, has been proposed. A chameleon field, characterised by a "chameleon mechanism" is an example of such a field nonminimally coupled to matter [7,8]. A consequence of this mechanism is that the mass of the scalar field is not a constant but rather changes with the ambient matter distribution. This change of properties of the field acts as a buffer in connection with the observational bounds set on the mass of the scalar fields coupled to the matter sector [9].

Chameleon scalar fields have many interesting features. For instance, a strongly coupled chameleon has the possibility of being detected by carefully designed experiments [10,11]. Possible effect of the chameleon field on the cosmic microwave background with possible observational imprints has been estimated by Davis, Schelpe and Shaw [12] and that on the rotation curve of galaxies by Burikham and Panpanich [13]. One very attractive feature of a chameleon field is that if it is coupled to an electromagnetic field [14] in addition to the fluid, the fine tuning of the initial conditions on the chameleon may be resolved to a large extent [15]. The remarkable features of the chameleon field theories are comprehensively summarised by Khoury [16].

Brax et al. used this chameleon field as a dark energy [17]. This kind of interaction between the dark matter and the dark energy was investigated in detail by Das, Corasaniti and Khoury [18] in the context of the present acceleration of the universe. It was also shown that with a chameleon field of this sort, it is quite possible to obtain a smooth transition from a decelerated to an accelerated expansion for the universe [19].

The success of the chameleon mechanism in explaining the current accelerated expansion and its lucrative properties which open up the possibility to evade a fine tuning of initial conditions, and its possible observational imprints inevitably attracted a lot of attention. The possibility of a scalar field nonminimally coupled to gravity, such as the Brans–Dicke scalar field, acting as a chameleon was discussed by Das and Banerjee [20]. Brans–Dicke scalar field acting as a chameleon with an infrared cut-off as that in the holographic models was discussed by Setare and Jamil [21]. The field profile of a chameleon was discussed by Tsujikawa, Tamaki and Tavakol [22].

The aim of the present work is to thoroughly investigate the stability criteria of the chameleon models in a spatially homogeneous and isotropic cosmology. There are two arbitrary functions of the chameleon field ϕ to start with, namely $V = V(\phi)$ and $f = f(\phi)$. Here V is the dark energy potential and f determines the coupling of the chameleon field with the matter sector. We broadly classify the functions into two categories, exponential and non-exponential. So there are four combinations in all. We investigate the conditions for having a stable solution for the evolution for each of these categories. We find that there are possibilities of finding a stable evolution scenario where the universe may settle into a phase of accelerated expansion. However, if both V and f are exponential functions of ϕ, the stability is very strongly dictated by the model parameters. In fact it is noted that in this latter case there is a possibility of a transient acceleration at the present epoch but the final stable configuration of the universe is that of a decelerated expansion.

The method taken up is the dynamical systems study. The field equations are written as an autonomous system and the fixed points are found out. A stable fixed point indicates a sink and thus marks the possible stable final configuration of the universe whereas an unstable fixed point, indicating a source, may describe the possible beginning of the evolution. Application of dynamical systems in cosmological problems, mostly for scalar field distributions, is already there in the literature [23]. For detailed discussions on some early work on such investigations, we refer to the monograph by Coley [24] (see also [25]).

The paper is organised as follows. Section 2 deals with a chameleon scalar field model in a spatially flat homogeneous and isotropic universe. In Section 3 the system of equations given in Section 2 are written as an autonomous system. This section also includes a brief discussion of the method of the stability analysis that is used in the present work. The actual stability analysis in the four categories as mentioned is given in Section 4. The fifth section presents an example of a chameleon field where the chameleon mechanism together with the constraints imposed by laboratory experiments does not have any contradiction with the cosmological requirement of the decelerated expansion entering into an accelerated phase at a redshift of 0.74 [26]. The sixth and the final section summarises and discusses the results.