Annals of Physics 356 (2015) 467–497

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Single-particle spectral density of the unitary Fermi gas: Novel approach based on the operator product expansion, sum rules and the maximum entropy method

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ARTICLE INFO

Article history:
Received 24 January 2015
Accepted 8 March 2015
Available online 15 March 2015

Keywords:
Unitary Fermi gas
Operator product expansion
Sum rules
Maximum entropy method

ABSTRACT

Making use of the operator product expansion, we derive a general class of sum rules for the imaginary part of the single-particle self-energy of the unitary Fermi gas. The sum rules are analyzed numerically with the help of the maximum entropy method, which allows us to extract the single-particle spectral density as a function of both energy and momentum. These spectral densities contain basic information on the properties of the unitary Fermi gas, such as the dispersion relation and the superfluid pairing gap, for which we obtain reasonable agreement with the available results based on quantum Monte-Carlo simulations.

1. Introduction

The unitary Fermi gas, consisting of non-relativistic fermionic particles of two species with equal mass, has been studied intensively during the last decade [1–3]. The growing interest in this system

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http://dx.doi.org/10.1016/j.aop.2015.03.007
0003-4916/

was prompted especially by the ability of tuning the interaction between different fermionic species in ultracold atomic gases through a Feshbach resonance by varying an external magnetic field. This technique allows one to bring the two-body scattering length of the two species to infinity and therefore makes it possible to study the unitary Fermi gas experimentally. Using photoemission spectroscopy, the measurement of the elementary excitations of ultracold atomic gases has in recent years become a realistic possibility [4,5]. Understanding these elementary excitations from a theoretical point of view is hence important and a number of studies devoted to this topic have already been carried out [6–8]. We will in this work propose a new and independent method for computing the single-particle spectral density of the unitary Fermi gas, which makes use of the operator product expansion (OPE).

The OPE, which was originally proposed in the late sixties independently by Wilson, Kadanoff and Polyakov [9–11], has proven to be a powerful tool for analyzing processes related to QCD (Quantum Chromo Dynamics), for which simple perturbation theory fails in most cases. The reason for this is the ability of the OPE to incorporate non-perturbative effects into the analysis as expectation values of a series of operators, which are ordered according to their scaling dimensions. Perturbative effects can on the other hand be treated as coefficients of these operators (the "Wilson-coefficients"). The OPE has specifically been used to study deep inelastic scattering processes [12] and has especially played a key role in the formulation of the so-called QCD sum rules [13,14].

In recent years, it was noted that the OPE can also be applied to strongly coupled non-relativistic systems such as the unitary Fermi gas [15–27]. Initially, the OPE was used to rederive some of the Tan-relations [28–30] in a natural way [15] and, for instance, to study the dynamic structure factor of unitary fermions in the large energy and momentum limit [20,23]. Furthermore, the OPE for the single-particle Green's function of the unitary Fermi gas was computed by one of the present authors [25] up to operators of momentum dimension 5, from which the single-particle dispersion relation was extracted. As the OPE is an expansion at small distances and times (or large momenta and energies), the result of such an analysis can be expected to give the correct behavior in the large momentum limit and is bound to become invalid at small momenta. The analysis of [25] confirmed this, but in addition somewhat surprisingly showed that the OPE is valid for momenta as small as the Fermi momentum k_F, where the OPE still shows good agreement with the results obtained from quantum Monte-Carlo simulations [7].

The purpose of this paper is to extend this analysis to smaller momenta, by making use of the techniques of QCD sum rules, which have traditionally been employed to study hadronic spectra from the OPE applied to Green's functions in QCD. Our general strategy goes as follows:

- Step 1: Construct OPE

  At first, we need to obtain the OPE for the single-particle Green's function $\mathcal{G}_\uparrow(k_0, \boldsymbol{k})$ in the unitary limit, which can be rewritten as an expansion of the single-particle self-energy $\Sigma_\uparrow(k_0, \boldsymbol{k})$. The subscript $\uparrow$ here represents the spin-up fermions. The main work of this step has already been carried out in [25]. $\Sigma_\uparrow(k_0, \boldsymbol{k})$ can be considered to be an analytic function on the complex plane of the energy variable k_0, with the exception of possible cuts and poles on the real axis. Considering the OPE at $T = 0$, with equal densities for both fermionic species ($n_\uparrow = n_\downarrow$) and taking into account operators up to momentum dimension 5, the only parameters appearing in the OPE are the Bertsch parameter and the contact density, which are by now well known from both experimental measurements [31–33] and theoretical quantum Monte-Carlo calculations [34,35].
- Step 2: Derive sum rules

  From the fact that the OPE is valid at large $|k_0|$ and the analytic properties of the self-energy, a general class of sum rules for Im $\Sigma_\uparrow(\omega, \boldsymbol{k})$ can be derived. In contrast to the complex k_0, ω here is a real parameter. These sum rules are relations between certain weighted integrals of Im $\Sigma_\uparrow(\omega, \boldsymbol{k})$ and corresponding analytical expressions that can be obtained from the OPE result (for details see Section 2):

$$D_\uparrow^{\mathrm{OPE}}(M, \boldsymbol{k}) = \int_{-\infty}^{\infty} d\omega \mathcal{K}(\omega, M) \mathrm{Im}\, \Sigma_\uparrow(\omega, \boldsymbol{k}). \tag{1}$$

  The kernel $\mathcal{K}(\omega, M)$ here must be an analytic function that is real on the real axis of ω and falls off to zero quickly enough at $\omega \to +\infty$, while M is some general parameter that characterizes