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Shifted one-parameter supersymmetric family of quartic asymmetric double-well potentials

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HIGHLIGHTS

- Quartic one-parameter DWs with an additional shift parameter are introduced.
- Anomalous localization feature of their zero modes is confirmed at different shifts.
- Razavy one-parameter DWs are also introduced and shown not to have this feature.

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ABSTRACT

Extending our previous work (Rosu, 2014), we define supersymmetric partner potentials through a particular Riccati solution of the form $F(x) = (x - c)^2 - 1$, where c is a real shift parameter, and work out the quartic double-well family of one-parameter isospectral potentials obtained by using the corresponding general Riccati solution. For these parametric double well potentials, we study how the localization properties of the two wells depend on the parameter of the potentials for various values of the shifting parameter. We also consider the supersymmetric parametric family of the first double-well potential in the Razavy chain of double well potentials corresponding to $F(x) = \frac{1}{2}\sinh 2x - 2\frac{(1+\sqrt{2})\sinh 2x}{(1+\sqrt{2})\cosh 2x+1}$, both unshifted and shifted, to test and compare the localization properties.

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1. Introduction

Anharmonic polynomial potentials, especially the quadratic–quartic double wells (DW), have been used paradigmatically in physics since many years and strengthened our understanding of the natural world at both realistic and fundamental levels. Quadratic–quartic DWs can be encountered in the literature as early as 1918 in the work of Duffing on nonlinear oscillations in such potentials [1]. In our times, dubbed as Mexican-hat potentials, they lie at the core of the origin of mass paradigm for elementary particles according to the Higgs mechanism, and many other important applications can be tracked in physics, such as semiconductor heterostructures, atom transfer in a scanning tunneling microscope, and the Bose–Einstein condensation.

In one-dimensional quantum mechanics, one way of introducing DWs is by using directly Riccati equations [2] or alternatively the Darboux transformations [3] and simple supersymmetric quantum mechanics [4]. In fact, all these methods are connected mathematically under the common idea of supersymmetry which originally occurred in field theory as a symmetry relating fermions and bosons. A perfect supersymmetry of the world would mean that each elementary particle would have a supersymmetric partner, which is either a fermion or a boson if the original particle is a boson or a fermion, respectively. Except for the spin, all the other quantum numbers of the superpartner particles are the same as well as their mass. Thus, the difference between them might be detected through some peculiar spin interactions which explains why no superpartner particle has been discovered yet. At the level of nonrelativistic quantum mechanics, supersymmetry means that the superpartner potentials are isospectral, i.e., if one knows the spectrum of one of them, the other has identical spectrum except perhaps the ground level and also their eigenfunctions are interrelated. Thus, there is not much hope to distinguish between the two superpartner potentials by some energy-changing processes and for many years people believed that supersymmetric quantum mechanics can be used only to extend the number of exactly solvable quantum problems. Nevertheless, recently there were proposals to look for experimental signatures of the level degeneracy due to supersymmetry in cold atoms in optical lattices [5–7], in arrays of waveguides [8], and in tunneling experiments involving a superconducting island coupled to a Josephson junction, the so-called Majorana Cooper-pair box [9].

However, another interesting class of DWs are the parametric DW potentials obtained in supersymmetric quantum mechanics through general solutions of Riccati equations. Their particular features are that they are related to the Abraham–Moses potentials in inverse scattering and they are strictly isospectral to the potentials from which they are constructed in the sense that even their ground state has the same energy as that of the original potential [10–12]. In our recent paper dedicated to the parametric supersymmetric potentials [13], we introduced a linear–quadratic–quartic class of one-parameter asymmetric DWs. In the limiting non-parametric case, such potentials occur as effective potentials at the tip of scanning tunneling microscope in the process of atom transfer during a voltage pulse [14]. The construction of this type of parametric potentials is based on the general Riccati solution obtained by the well-known Bernoulli ansatz from a particular Riccati solution of the form $F(x) = (x-1)^2 - 1$. In that case, we noted the interesting feature that the parameter of the potential controls the heights of the localization probability in the two wells, and for certain values of the parameter the height of the localization probability can be higher in the shallower well. In the following the parameter range where this happens will be called *anomalous localization region* (ALR), while the remaining range is the regular localization region.

The main goal of the present work is to extend the study of these quartic DWs with ALR by employing the same particular quadratic Riccati solutions endowed with an arbitrary real shift parameter c, namely $F(x) = (x-c)^2 - 1$. In other words, we will investigate what happens with the ALR effect when the one-parameter supersymmetric construction is displaced along the axis by c units. In addition, we will use one case of parametric DW from the Razavy chain of supersymmetric potentials to examine the issue of the generality of this interesting localization property.

2. One-parameter supersymmetric isospectral potentials

First, we briefly present the mathematical scheme of SUSY QM that we used in [13]. The following two Schrödinger equations (over the paper, the prime notation is used for the derivatives with