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A practical introduction to tensor networks: Matrix product states and projected entangled pair states

Román Orús *

Institute of Physics, Johannes Gutenberg University, 55099 Mainz, Germany

HIGHLIGHTS

- A practical introduction to selected aspects of tensor network methods is presented.
- We provide analytical examples of MPS and 2d PEPS.
- We provide basic aspects on several numerical methods for MPS and 2d PEPS.
- We discuss a number of applications of tensor network methods from a broad perspective.

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ABSTRACT

This is a partly non-technical introduction to selected topics on tensor network methods, based on several lectures and introductory seminars given on the subject. It should be a good place for newcomers to get familiarized with some of the key ideas in the field, specially regarding the numerics. After a very general introduction we motivate the concept of tensor network and provide several examples. We then move on to explain some basics about Matrix Product States (MPS) and Projected Entangled Pair States (PEPS). Selected details on some of the associated numerical methods for 1*d* and 2*d* quantum lattice systems are also discussed.

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* Tel.: +49 6131 39 204 61.
E-mail address: roman.orus@uni-mainz.de.

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1. Introduction

During the last years, the field of Tensor Networks has lived an explosion of results in several directions. This is specially true in the study of quantum many-body systems, both theoretically and numerically. But also in directions which could not be envisaged some time ago, such as its relation to the holographic principle and the AdS/CFT correspondence in quantum gravity [1,2]. Nowadays, Tensor Networks is rapidly evolving as a field and is embracing an interdisciplinary and motivated community of researchers.

This paper intends to be an introduction to selected topics on the ever-expanding field of Tensor Networks, mostly focusing on some practical (i.e. algorithmic) applications of Matrix Product States and Projected Entangled Pair States. It is mainly based on several introductory seminars and lectures that the author has given on the topic, and the aim is that the unexperienced reader can start getting familiarized with some of the usual concepts in the field. Let us clarify now, though, that we do not plan to cover *all* the results and techniques in the market, but rather to present some insightful information in a more or less comprehensible way, sometimes also trying to be intuitive, together with further references for the interested reader. In this sense, this paper is not intended to be a complete review on the topic, but rather a useful manual for the beginner.

The text is divided into several sections. Section 2 provides a bit of background on the topic. Section 3 motivates the use of Tensor Networks, and in Section 4 we introduce some basics about Tensor Network theory such as contractions, diagrammatic notation, and its relation to quantum many-body wave-functions. In Section 5 we introduce some generalities about Matrix Product States (MPS) for 1d systems and Projected Entangled Pair States (PEPS) for 2d systems. Later in Section 6 we explain several strategies to compute expectation values and effective environments for MPS and PEPS, both for finite systems as well as systems in the thermodynamic limit. In Section 7 we explain generalities on two families of methods to find ground states, namely variational optimization and imaginary time evolution. Finally, in Section 8 we provide some final remarks as well as a brief discussion on further topics for the interested reader.