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Complete particle-pair annihilation as a dynamical signature of the spectral singularity

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HIGHLIGHTS

- We investigate the physical relevance of a spectral singularity.
- The two-particle collision leads to amplitude-reduction of the wave function.
- There is a singularity spectrum which leads to complete particle-pair annihilation.
- Complete particle-pair annihilation can only occur for two distinguishable bosons and singlet fermions.
- Pair annihilation provides a detection method of the spectral singularity in the experiment.

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ABSTRACT

Motivated by the physical relevance of a spectral singularity of interacting many-particle system, we explore the dynamics of two bosons as well as fermions in one-dimensional system with imaginary delta interaction strength. Based on the exact solution, it shows that the two-particle collision leads to amplitude-reduction of the wave function. For fermion pair, the amplitude-reduction depends on the spin configuration of two particles. In both cases, the residual amplitude can vanish when the relative group velocity of two single-particle Gaussian wave packets with equal width reaches the magnitude of the interaction strength, exhibiting complete particle-pair annihilation at the spectral singularity.

1. Introduction

Non-Hermitian operator has been introduced phenomenologically as an effective Hamiltonian to fit experimental data in various fields of physics [1–5]. In spite of the important role played by the non-

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Hermitian operator in different branches of physics, it has not been paid due attention by the physics community until the discovery of non-Hermitian Hamiltonians with parity–time symmetry, which have a real spectrum [6]. It has boosted the research on the complex extension of quantum mechanics on a fundamental level [7–17]. Non-Hermitian Hamiltonian can possess peculiar feature that has no Hermitian counterpart. A typical one is the spectral singularity (or exceptional point for finite system), which is a mathematic concept. It has gained a lot of attention recently [18–26], motivated by the possible physical relevance of this concept since the pioneer work of Mostafazadeh [27].

The majority of previous works focus on the non-Hermitian system arising from the complex potential, mean-field nonlinearity [20,28–37] as well as imaginary hopping integral [38]. In this paper, we investigate the physical relevance of the spectral singularities for the non-Hermitian interacting many-particle system. The non-Hermiticity arises from the imaginary interaction strength. For two-particle case, the exact solution shows that there exist a series of spectral singularities, forming a spectrum of singularity associated with the central momentum of the two particles. We consider dynamics of two bosons as well as fermions in one-dimensional system with imaginary delta interaction strength. It shows that the two-particle collision leads to amplitude-reduction of the wave function. For fermion pair, the amplitude-reduction depends on the spin configuration of two particles. Remarkably, in both cases, the residual amplitude can vanish only when the relative group velocity of two single-particle Gaussian wave packets with equal width reaches the magnitude of the interaction strength. This phenomenon of complete particle-pair annihilation is the direct result of the spectral singularity. We also discuss the complete annihilations of a singlet fermion pair and a maximally two-mode entangled boson pair based on the second quantization formalism.

This paper is organized as follows. In Section 2, we present the model Hamiltonian and the exact solution. In Section 3, we construct the local boson pair initial state as initial state which is allowed to calculate the time evolution. Based on this, we reveal the connection between the phenomenon of complete pair annihilation and the spectral singularity. In Section 4, we extend our study to a singlet fermion pair and a maximally two-mode entangled boson pair based on the second quantization formalism. Finally, we give a summary in Section 5.

2. Hamiltonian and solutions

We start with a one-dimensional two-distinguishable particle system with imaginary delta interaction. The solution can be used to construct the eigenstates of two-fermion and boson systems. The Hamiltonian has the form

$$H_{2\mathrm{p}} = -\frac{1}{2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) - i2\gamma\delta\left(x_1 - x_2\right) \tag{1}$$

where $\gamma > 0$ and we use dimensionless units $e = \hbar = m = 1$ for simplicity.

Introducing new variables R and r, where

$$R = (x_1 + x_2)/2, \qquad r = x_1 - x_2, \tag{2}$$

we obtain the following Hamiltonian:

$$H_{2\mathrm{p}} = H_{\mathrm{R}} + H_{\mathrm{r}}, \tag{3}$$

with

$$H_{\mathrm{R}} = -\frac{\partial^2}{4\partial R^2}, \qquad H_{\mathrm{r}} = -\frac{\partial^2}{\partial r^2} - i2\gamma\delta(r). \tag{4}$$

Here R is the center-of-mass coordinate and r is the relative coordinate. The Hamiltonian is separated into a center-of-mass part and a relative part, and can be solvable exactly.

The eigenfunctions of the center-of-mass motion H_{R} are simply plane waves, while the Hamiltonian H_{r} is equivalent to that of a single-particle in an imaginary delta-potential, which has been exactly