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Gapless excitations of axially symmetric vortices in systems with tensorial order parameter

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ABSTRACT

We extend the results of previous work on vortices in systems with tensorial order parameters. Specifically, we focus our attention on systems with a Ginzburg–Landau free energy with a global $U(1)_P \times SO(3)_S \times SO(3)_L$ symmetry in the phase, spin and orbital degrees of freedom. We consider axially symmetric vortices appearing on the spin–orbit locked $SO(3)_{S+L}$ vacuum. We determine the conditions required on the Ginzburg–Landau parameters to allow for an axially symmetric vortex with off diagonal elements in the order parameter to appear. The collective coordinates of the axial symmetric vortices are determined. These collective coordinates are then quantized using the time dependent Ginzburg–Landau free energy to determine the number of gapless modes propagating along the vortex.

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1. Introduction

In a previous publication [1], we considered emergent collective coordinates on vortices in systems with a Ginzburg–Landau free energy with a $U(1)_P \times SO(3)_S \times SO(3)_L$ symmetry group. Here, $U(1)_P$ represents the phase symmetry of the order parameter, and $SO(3)_{S,L}$ are the spin and orbital rotation groups. In particular, we considered vortices appearing in the bulk spin–orbit locked $SO(3)_{S+L}$ vacuum, much like the B-phase of superfluid ^{3}He. In that work, we provided a method for determining the type and number of collective coordinates appearing on mass vortices in the $SO(3)_{S+L}$ vacuum. This was

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performed under the restriction that the 3×3 matrix order parameter contained only its diagonal ($\delta_{\mu i}$) and antisymmetric off-diagonal ($\varepsilon_{\mu ik}\chi_k$) elements, setting its symmetric-traceless part to zero. This restriction allowed the calculations to be carried out with relative ease while still providing enough depth to illustrate the development of non-Abelian collective coordinates. We were also able to illustrate the interactions of the non-Abelian collective coordinates with the well studied translational degrees of freedom leading to the familiar Kelvin modes [2–9]. The presentation of that work was purely classical and we made no attempt to discuss time dependence or quantization. Including quantization may alter the number of gapless excitations determined from the naive counting of collective coordinates. In this work, we aim to continue the analysis and discuss vortices that are more closely related to those studied both experimentally [6–8], and theoretically [10,11], in superfluid ^{3}He–B. Specifically, we will discuss vortices respecting an axial $O(2)$ symmetry, which we will define below. We will also explore the effects of time dependence and quantization on the gapless excitations of axial symmetric vortices.

Condensed matter systems with tensorial order parameters such as superfluid ^{3}He have drawn attention from the high-energy physics community [12,13]. The non-Abelian group structure inherent to such condensed matter systems suggests interesting parallels with high energy systems with similar non-Abelian symmetries. One example of this parallel behavior is the case of low energy excitations of vortices in systems similar to superfluid ^{3}He–B, which can be compared with the low energy excitations of Abrikosov–Nielsen–Olesen-like (ANO) flux tubes [14,15] appearing in Yang–Mills theories [16–21]. Under a particular choice of the Ginzburg–Landau free energy parameters, the vortices in such condensed matter systems develop internal non-Abelian collective excitations whose low energy dynamics follow a non-linear $O(3)$ sigma model [22]. The same $O(3)$ sigma model emerges for the collective modes of the flux tubes strings in Yang–Mills theories [23–25]. This example is illustrating the universality of effective field theories in condensed matter, high energy, and cosmological systems [12].

The model we consider is described by a Ginzburg–Landau free energy with a continuous $U(1)_P \times SO(3)_S \times SO(3)_L$ symmetry. This structure appears near a critical temperature in the BCS theory of orbital P-wave pairing of identical fermions [26]. To satisfy the anti-symmetric condition, the fermion pair must be in a spin triplet state. Thus, the order parameter describing the system is a complex 3×3 matrix $e_{\mu i}$, where μ and i describe the spin and orbital degrees of freedom, respectively. This is what happens, in particular, in superfluid ^{3}He [27–29]. We hasten to point out that the models we consider below are inspired by considerations of superfluid ^{3}He. However, aside from initial inspirations and similarities in terminology, we do not wish to invite this comparison any further in what follows.

The free energy, from which we start, contains an enhancement of the orbital rotation symmetry whereby the internal and external rotations will be considered separately (see the example in [30]). By external coordinate rotation, we are referring to a transformation resulting from a rotation of the coordinate system without a corresponding rotation of the orbital index. In this case, we can consider the orbital index as an internal degree of freedom. Thus, the continuous symmetry of the model is

$$\mathcal{G} = U(1)_P \times SO(3)_S \times SO(3)_{L_{\text{int}}} \times SO(3)_{L_{\text{ext}}} \times T \tag{1}$$

where the enhanced $SO(3)_L \to SO(3)_{L_{\text{int}}} \times SO(3)_{L_{\text{ext}}}$ is explicitly written. The group T represents the translational symmetry. We refer to the internal part of $\mathcal{G}$ as $G = U(1)_P \times SO(3)_S \times SO(3)_{L_{\text{int}}}$. This type of symmetry can be approximately achieved in an ultra-cold fermi gas with p-wave pairing. Additionally, this situation is reminiscent of the theory of elasticity where an unphysical vanishing of the bulk modulus leads to an enhanced symmetry of rotations $O(2) \to O(2) \times O(2)$. This leads to the equivalence of scale and conformal transformations [31]. In this work, it will be necessary to consider the entire symmetry group including both the internal and external symmetries (see Figs. 1 and 2).

In the vacuum, a spontaneous breaking of the internal symmetry G to a spin–orbit locked symmetry

$$G \to H = SO(3)_{S+L_{\text{int}}} \tag{2}$$

occurs, similar to the mechanism of color–flavor locking in color superconductivity [32,33]. This vacuum has a degeneracy given by

$$G/H = U(1)_P \times SO(3)_{S-L_{\text{int}}}. \tag{3}$$

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