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Monotonically increasing functions of any quantum correlation can make all multiparty states monogamous

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ABSTRACT

Monogamy of quantum correlation measures puts restrictions on the sharability of quantum correlations in multiparty quantum states. Multiparty quantum states can satisfy or violate monogamy relations with respect to given quantum correlations. We show that all multiparty quantum states can be made monogamous with respect to all measures. More precisely, given any quantum correlation measure that is non-monogamic for a multiparty quantum state, it is always possible to find a monotonically increasing function of the measure that is monogamous for the same state. The statement holds for all quantum states, whether pure or mixed, in all finite dimensions and for an arbitrary number of parties. The monotonically increasing function of the quantum correlation measure satisfies all the properties that are expected for quantum correlations to follow. We illustrate the concepts by considering a thermodynamic measure of quantum correlation, called the quantum work deficit.

1. Introduction

Sharing of quantum correlations among many parties is known to play an important role in quantum phenomena, ranging from quantum communication protocols [1–4] to cooperative events

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in quantum many-body systems [5,6]. It is therefore important to conceptualize and quantify quantum correlations, for which investigations are usually pursued in two directions, viz. the entanglement–separability [7] and the information-theoretic [8] ones. Any such measure of quantum correlation is expected to satisfy a monotonicity (precisely, non-increasing) under an intuitively satisfactory set of local quantum operations.

For a quantum state which is shared between more than two parties, one may expect that all the measures of quantum correlation would additionally follow a monogamy property [9–12], which restricts the sharability of quantum correlations among many parties. In the case of three parties, say, Alice, Bob and Charu, monogamy of a measure, $\mathcal{Q}$, says that the sum, $\mathcal{Q}_{AB} + \mathcal{Q}_{AC}$, of quantum correlations of the two-party local states between the Alice–Bob and the Alice–Charu pairs, should not exceed the quantum correlation, $\mathcal{Q}_{A:BC}$, of Alice with Bob and Charu taken together. Alice is therefore alloted a special status, and is called the "nodal observer". If the tripartite state, shared between the three parties, Alice, Bob, and Charu, is symmetric under exchange of particles, then any of the three parties in the monogamy relation can act as the nodal observer. However, if the state under consideration is non-symmetric under interchange of particles, then we allot the status of the nodal observer to the party i that minimizes the monogamy expression $\mathcal{Q}_{i:jk} - \mathcal{Q}_{ij} - \mathcal{Q}_{ik}$, with i, j, k being chosen from Alice, Bob, and Charu, and with no two of i, j, k being equal. Let us mention however that our results hold with other choices of the nodal observer also. The concept of monogamy has also been carried over to more than two extra-nodal observers. Classical correlations certainly do not satisfy a monogamy constraint [12]. The monogamous nature of quantum correlations plays a key role in the security of quantum cryptography [13]. Moreover, monogamy of quantum correlations has recently been used to study frustrated spin systems [14]. Surprisingly however, there are important and useful entanglement measures that do not satisfy monogamy for certain multiparty quantum states, an example being the entanglement of formation [10], which quantifies the amount of entanglement required for preparation of a given bipartite quantum state. Nevertheless, it was found that for multiqubit systems, the concurrence squared [15], a monotonically increasing function of the entanglement of formation is monogamous [9–12]. Similarly, the square of concurrence and entanglement of formation are monogamous for arbitrary multiqubit systems [16], although concurrence and entanglement of formation themselves are not so. Recently, it was shown that the information-theoretic quantum correlation measure, quantum discord [17,18], can violate monogamy [19–22] (cf. [23,24]), and again a monotonically increasing function of the quantum discord satisfies monogamy for three-qubit pure states [25].

In this paper, we show that if any bipartite quantum correlation measure, of an arbitrary number of parties in arbitrary finite dimensions, is non-increasing under loss of a part of a local subsystem, any multiparty quantum state is either already monogamous with respect to that measure or an increasing function of the bipartite measure can make it so. Note that the result holds for both pure and mixed states. It is interesting to note that the increasing function also satisfies all the properties for being a measure of quantum correlation, which include monotonicity under local operations and vanishing for "classically correlated" states (which is the set of separable states for measures of entanglement). Moreover we show that the function can always be chosen to be reversible, so that there is no loss of information in applying the function on the parent quantum correlation [7,26]. To illustrate the result, we show that although the quantum work-deficit [27], an information-theoretic quantum correlation measure, violates monogamy even for three-qubit pure states, the states become monogamous when one considers integer powers of the measure. In stark contrast to what happens for concurrence and quantum discord, we show that for the three-qubit generalized W states [28,29], the fourth power of quantum work-deficit is required to obtain monogamy for these states. In case of arbitrary three-qubit W-class states [28,29] and the GHZ-class states [30,29], to obtain monogamy of quantum work-deficit, one requires higher polynomials. We also find that three-qubit pure states that are monogamous with respect to quantum discord are also so with respect to quantum work-deficit.

2. Turning non-monogamous multisite quantum states into monogamous ones

Let $\mathcal{Q}$ be a quantum correlation measure that is defined for arbitrary bipartite states (pure or mixed) in arbitrary finite dimensions. Consider a three-party quantum state (pure or mixed), ϱ_{ABC}, in arbitrary