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Covariant energy–momentum and an uncertainty principle for general relativity

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HIGHLIGHTS

- We present a totally invariant spacetime energy expression for general relativity incorporating the contribution from gravity.
- Demand for the general expression to reduce to the Tolman integral for stationary systems supports the Ricci integral as energy–momentum.
- Localized energy via the Ricci integral is consistent with the energy localization hypothesis.
- New localized energy supports the Bonnor claim that the Szekeres collapsing dust solutions are energy-conserving.
- Suggest the Heisenberg Uncertainty Principle be generalized in terms of spacetime energy–momentum in strong gravity extreme.

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ABSTRACT

We introduce a naturally-defined totally invariant spacetime energy expression for general relativity incorporating the contribution from gravity. The extension links seamlessly to the action integral for the gravitational field. The demand that the general expression for arbitrary systems reduces to the Tolman integral in the case of stationary bounded distributions, leads to the matter-localized Ricci integral for energy–momentum in support of the energy localization hypothesis. The role of the observer is addressed and as an extension of the special relativistic case, the field of observers comoving with the matter is seen to compute the intrinsic global energy of a system. The new localized energy supports the Bonnor claim that the Szekeres collapsing dust solutions are energy-conserving. It is suggested that in the extreme of strong

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gravity, the Heisenberg Uncertainty Principle be generalized in terms of spacetime energy–momentum.

1. Introduction

The unresolved problems of energy and the issue of its localization within general relativity were debated in the early years of the theory's formulation. Given their fundamental importance, it is understandable that they have remained subjects of considerable interest even up to recent times. By way of a brief history, in the 1920s, some authors including Einstein and Eddington held the view that while one could work usefully with energy in the traditional sense as a global concept in general relativity, no satisfactory meaning could be attached to its localization, a situation unprecedented in all the rest of physics. Their belief was based on the manner in which energy–momentum for the gravitational field was incorporated into general relativity. Rather than having a *bona fide* energy–momentum tensor T_i^k[1] in general relativity to incorporate energy–momentum as in the rest of physics, these authors relied upon an energy–momentum pseudotensor t_i^k, first introduced by Einstein, to play the equivalent role for gravity. Unlike true tensors, this pseudotensor could be made to vanish at any preassigned point by an appropriate transformation of coordinates, rendering its status rather nebulous. The pseudotensor was introduced in order to incorporate a global energy and momentum into general relativity, a necessary exercise, it was felt, because gravity had not lent itself to inclusion in the energy–momentum tensor T_i^k as it had for all other fields. All fields other than the gravitational field incorporated themselves into the energy–momentum tensor, and global energy–momentum conservation followed naturally through the vanishing of the ordinary divergence of the energy–momentum tensor,[2]

$$\partial T_i^k/\partial x^k = T_{i,k}^k = 0. \tag{1}$$

By integrating (1) over a given 3-volume and applying the Gauss divergence theorem, one readily expresses the time-rate of change of energy and momentum in the given 3-volume as accounted for by the flux of energy and momentum over the bounding 2-surface of this 3-volume.

However, in general relativity, (1) no longer applies. Rather, it is replaced by the vanishing *covariant* divergence of the energy–momentum tensor, viz.[3]

$$T_{i;k}^k = 0. \tag{2}$$

Eq. (2) is the local expression for energy–momentum conservation in general relativity. Through the covariant derivative, it brings the metric and hence gravity into the conservation statement. However, with this new form, one can no longer write the integral conservation laws as was the case previously in special relativity without an essential modification, the introduction of the aforementioned pseudotensor t_i^k. When this is done, (2) is re-expressed as a vanishing ordinary divergence,

$$(\sqrt{-g}(T_i^k + t_i^k))_{,k} = 0 \tag{3}$$

where g is the determinant of the metric tensor g_{ik}.

Through the years, other pseudotensors performing the same function as that of Einstein's pseudotensor were introduced but they all carried the stigma of being non-covariant objects. In addition, they were not symmetric and hence did not lend themselves to forming an angular momentum construct as does the symmetric energy–momentum tensor T^{ik} of special relativity. Landau and Lifshitz [1]

1 Latin indices range from 0 to 3 and Greek indices range from 1 to 3. We use units in which $G = c = 1$.
2 Repeated indices imply summation.
3 A semi-colon denotes covariant differentiation.