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Comparison of different pairing fluctuation approaches to BCS–BEC crossover

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ABSTRACT

The subject of BCS–Bose–Einstein condensation (BEC) crossover is particularly exciting because of its realization in ultracold atomic Fermi gases and its possible relevance to high temperature superconductors. In this paper we review the body of theoretical work on this subject, which represents a natural extension of the seminal papers by Leggett and by Nozières and Schmitt-Rink (NSR). The former addressed only the ground state, now known as the "BCS-Leggett" wave-function, and the key contributions of the latter pertain to calculations of the superfluid transition temperature T_c. These two papers have given rise to two main and, importantly, distinct, theoretical schools in the BCS–BEC crossover literature. The first of these extends the BCS-Leggett ground state to finite temperature and the second extends the NSR scheme away from T_c both in the superfluid and normal phases. It is now rather widely accepted that these extensions of NSR produce a different ground state than that first introduced by Leggett. This observation provides a central motivation for the present paper which seeks to clarify the distinctions in the two approaches. Our analysis shows how the NSR-based approach views the bosonic contributions more completely but treats the fermions as "quasi-free". By contrast, the BCS-Leggett based approach treats the fermionic contributions more completely but treats the bosons as "quasi-free". In a related fashion, the NSR-based schemes approach the crossover between BCS and BEC by starting from the BEC limit and the BCS-Leggett based scheme approaches this crossover by starting from the BCS limit. Ultimately, one would like to combine these two schemes. There are, however, many difficult problems to sur-

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mount in any attempt to bridge the gap in the two theory classes. In this paper we review the strengths and weaknesses of both approaches. The flexibility of the BCS-Leggett based approach and its ease of handling make it widely used in $T = 0$ applications, although the NSR-based schemes tend to be widely used at $T \neq 0$. To reach a full understanding, it is important in the future to invest effort in investigating in more detail the $T = 0$ aspects of NSR-based theory and at the same time the $T \neq 0$ aspects of BCS-Leggett theory.

1. Introduction

The subject of BCS–Bose–Einstein condensation (BEC) crossover has recently become an extremely active research area. This is due principally to the discovery [1–10] of superfluid phases in ultracold atomic Fermi gases which exhibit this crossover. Adding to the importance of this work is the view espoused by a number of theorists [11–15] that the high temperature superconductors are midway between BCS and BEC. Now, with an unambiguous realization of this scenario in the fermionic superfluids, one has the opportunity to investigate this physical picture more closely and, it is hoped, gain insight into the cuprate superconductors. Equally exciting is the opportunity to generalize, and in the process, gain insight into what is arguably the paradigm for all theories in condensed matter physics: Bardeen-Cooper-Schrieffer (BCS) theory. For all these reasons a large number of variants of BCS–BEC crossover theory have been suggested in the literature. It is the purpose of the present paper to present an overview of two main classes of theories, discussing their strengths and weaknesses. Contrasting and comparing different approaches will, hopefully, point to new directions for future theoretical and experimental research.

Initial theoretical work [16,17] on the subject of BCS–BEC crossover focused on a ground state which was shown to be the same as that proposed by Bardeen, Cooper, and Schrieffer, when it is extended to accommodate a continuous evolution from BCS to BEC. We call this the "BCS-Leggett" state. Here the fermionic chemical potential μ is solved self-consistently as the attractive interaction strength is varied. In this way it became clear that the BCS trial wavefunction was far more general than was originally thought. Somewhat later, Noziéres and Schmitt-Rink (NSR) [18] presented a scheme for calculating the superfluid transition temperatures T_c, which made the case that the evolution from BCS to BEC was again continuous at finite temperature.

The discovery of high temperature superconductivity and the observation that their coherence length ξ (or equivalently pair size) was anomalously small led Lee and Friedberg to argue that one should include bosonic degrees of freedom in addressing high T_c superconductors [19,20]. These authors introduced the "boson-fermion" model almost immediately after the discovery of cuprate superconductivity. In a similar vein, Randeria [11] proposed that the NSR scheme might be directly applicable to these exciting new materials. Subsequently other theorists have applied this BCS–BEC crossover scenario to the high T_c cuprates [21–23,15]. Additional support has come from the experimental condensed matter community among whom a number [24–27] have presented data which can be interpreted within this picture. Adding to the enthusiasm is the observation of a ubiquitous (albeit controversial) "pseudogap" phase [28,12,13] in the underdoped cuprates, which was argued [29,30] to be consistent with a BCS–BEC crossover scenario.

The characterization of pseudogap effects associated with BCS–BEC crossover was, in fact, a crucial step. It was first recognized that one should distinguish the pair formation temperature T^* from the condensation temperature T_c [31,11]. That the magnetic properties of the normal phase of a superconductor in the temperature regime between T_c and T^* would be anomalous was pointed out on the basis of numerical calculations, on a two-dimensional (2D) lattice. Here it was found that the spin susceptibility was depressed at low temperatures [32] and this depression was associated with a "spin gap" which is to be distinguished [29] from a pseudogap, which affects the "charge channel" as well.