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Callan–Symanzik method for *m*-axial Lifshitz points

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ABSTRACT

We introduce the Callan–Symanzik method in the description of anisotropic as well as isotropic Lifshitz critical behaviors. Renormalized perturbation theories are defined by normalization conditions with nonvanishing masses and at zero external momenta. We prove the multiplicative renormalizability of the field-theoretic formulation at the critical dimension. The orthogonal approximation is employed to obtain the critical indices η_{L2}, ν_{L2}, η_{L4} and ν_{L4} diagrammatically at least up to two-loop order in the anisotropic criticalities. This approximation is also utilized to compute the exponents η_{L4} and ν_{L4} in the isotropic case. Furthermore, we compute those exponents exactly for the isotropic behaviors at the same loop order. The results obtained for all exponents are in perfect agreement with those previously derived in the massless theories renormalized at nonzero external momenta.

1. Introduction

Renormalized perturbative expansion of a $\lambda\phi^4$ field theory is the natural mathematical setting in statistical mechanics to determine the critical properties taking place in second order phase transitions. For ordinary critical systems, the theory is defined on a d-dimensional Euclidean space whose bare Lagrangian density has quadratic derivatives of the field (order parameter) as kinetic term. These criticalities can be generalized by adding certain combinations of m quartic derivatives of the field to the kinetic term of the bare Lagrangian density. In fact, when the m coefficients of the usual quadratic derivatives of the field vanish, the Lagrangian density describes the so-called m-axial Lifshitz critical behaviors [1–3]. Physically, this sort of critical behavior describes systems with competing interactions, with a wide range of applications in actual physical systems. Among others, examples are

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encountered in particular types of polymers [4,5], liquid crystals [6], microemulsions [7], ferroelectrics [8], high-T_c superconductors [9] and magnetic compounds and alloys [10,11]. In addition, it has also been studied in the context of quantum phase transitions [12].

Using the language of magnetic systems in the simplest situation ($m = 1$), the bare Lagrangian describes a certain lattice model, the so-called ANNNI model [3] which consists of an Ising model with ferromagnetic exchange forces between nearest neighbors as well as antiferromagnetic interactions between second neighbors along just one space direction. The confluence of the ferromagnetic, paramagnetic and modulated phases define the uniaxial Lifshitz point. The antiferromagnetic couplings can then be generalized to include m space directions originating an m-axial Lifshitz point. The competition among ferro- and antiferromagnetic couplings of the spins in the model induces the vanishing of the m terms in the quadratic derivatives of the field keeping, however, the quartic derivatives along the competing directions. The m higher derivative terms will generate space anisotropy whenever $m < d$ (with two inequivalent space directions namely, the $(d - m)$-dimensional noncompeting subspace and the m-dimensional competing axes), whereas if $m = d$ (close to 8) there is only one isotropic space.

Renormalization group and ϵ-expansion ideas ($\epsilon = 4 - d$) were originally introduced to provide the perturbative determination of critical exponents using diagrammatic methods in momenta space [13]. After that, a reformulation of this original method incorporating conventional field theory arguments was devised in order to determine the critical exponents using a massless theory renormalized at nonzero external momenta [14,15]. Within this procedure, many theoretical investigations to study m-axial Lifshitz critical properties have been carried out [16–19]. More recently another alternative to compute critical exponents for m-axial Lifshitz critical behaviors has been proposed. It consists of the combination of unconventional renormalization group arguments (with two independent scale transformations for each subspace in the anisotropic cases [20]) together with a completely analytical solution of Feynman integrals by the introduction of the orthogonal approximation [21]. This approximation is the most general one consistent with homogeneity of the diagrams in the external momenta. It can be utilized to resolve loop integrals in the anisotropic and isotropic cases. Furthermore, it has been generalized to include arbitrary types of competing interactions [22]. In particular, the isotropic exponents for these generic universality classes were also computed exactly and a remarkable agreement with the exponents obtained via the generalized orthogonal approximation has been discovered [23]. In this framework, the critical exponents and all universal quantities reduce to the Ising-like amounts when the competing interactions are turned off [21–23].

A different trend to the above methods is to formulate the problem in a massive theory. This permits to comprehend under what conditions the scale invariance of the solution to the renormalization group equation for the renormalized 1PI vertex parts is guaranteed. In addition, this formulation might be convenient to treat renormalization issues of certain types of higher derivative quantum field theories and it is a healthy test for the universality hypothesis, which guarantees that universal quantities (e.g., the critical exponents) do not depend on the scheme of renormalization employed.

For ordinary critical systems, the asymptotic scale invariance of the theory in $d = 4 - \epsilon$ was established more rigorously after the first developments [13] through the use of Callan–Symanzik equations [24,25] in the explicit computation of the critical exponents for a massive theory [26]. However, the same approach has not been applied for investigating more general critical systems so far. In fact we would like to know if such a method could be adapted to critical systems exhibiting more than one characteristic length scale, such as the m-fold Lifshitz critical behavior. It is the purpose of the present work to show that this aim can be achieved, following closely the arguments put forth in [26]. Here, we derive the critical exponents η_{L2} and ν_{L2} perpendicular to the competing axes, as well as their counterpart η_{L4} and ν_{L4} along the competition directions in the massive case by introducing an appropriate Callan–Symanzik formalism with two independent mass scales in the bare Lagrangian. They correspond to the correlation lengths ξ_{L2} and ξ_{L4} present in the anisotropic critical behaviors. The massive theories are renormalized at zero external momenta. A similar treatment will be shown to be valid for the isotropic behaviors with only one mass scale. Consequently, we derive diagrammatically the critical exponents characterizing the anomalous dimensions of the field ϕ and composite operator ϕ^2, respectively, within an ϵ_L-expansion at least at two-loop order using Feynman's path integral method in momentum space.