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Chern–Simons and WZW anomaly cancelations across dimensions

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ABSTRACT

The WZW functional in $D = 4$ can be derived directly from the Chern–Simons functional of a compactified $D = 5$ gauge theory and the boundary fermions it supplants. A simple pedagogical model based on $U(1)$ gauge groups illustrates how this works. A bulk-boundary system with the fermions eliminated manifestly evinces anomaly cancelations between CS and WZW terms.

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1. Introduction

In this paper we illustrate in a simple scheme how the full Wess–Zumino–Witten (WZW) functional [1,2] of a gauged chiral Lagrangian in $D = 4$ arises out of a pure gauge theory of quark flavor in compactified $D = 5$. This model, based upon a $U(1)_L \times U(1)_R$ flavor symmetry, discussed in [3], mimics the chiral structure of QCD, and was used to clarify how the counterterm structure of the WZW functional arises in a parity-asymmetric gauging, such as in the Standard Model.

Here, however, we use it to illustrate how the generic features of the holographic origin of the chiral WZW term arise, where the $D = 4$ mesons emerge out of the Wilson line over the bulk gauge field, A_5. The general gauged WZW functional structure for $U(N) \times U(N)$ has been studied previously using deconstruction [4], while the general gauged Kaymakcalan, Rajeev, and Schechter (KRS) action [5] has been derived from continuum compactification of a pure $SU(N)$ Yang–Mills theory in detail in [6]. The present work is intended, in part, to clarify the approach and results of [4,6].

We construct a manifestly $U(1)$ gauge-invariant theory in $D = 5$. The gauge fields propagate in the bulk, with chiral quarks attached to chiral boundaries (branes), with L (R) located at $x^5 = 0$ ($x^5 = R$), respectively. The quarks are chirally delocalized in $D = 5$ [7], and their chiral anomalies [8] are nonzero on their respective boundaries, but would otherwise cancel if the boundaries were merged.

The boundary conditions on the $D = 5$ gauge fields are subject to a minimal set of constraints: (I) there exists a massless physical A_5 zero mode, (or, more properly, a nontrivial Wilson line spanning the bulk) which can be identified with chiral mesons; and (II) there exists a tower of KK modes of the gauge fields, which is sufficiently rich, such that independently valued combinations of these fields exist on the boundary branes.

Much of what we cover in our pedagogical model is expected to apply to any theory of new physics in extra dimensions which satisfies (I–II) with chiral delocalization, including AdS $D = 5$ models. Irrespective of the specific $D = 5$ geometry, our low-energy effective theory results are holographic, *i.e.*, they are determined at the boundary, as the integrands in the bulk involving the lower KK-modes are mostly *exact* expressions. Since the theory we consider is arranged to be anomaly free, the resulting effective action contains both the holographic WZW, and a dual effective interaction in the bulk. This latter bulk interaction takes the form of a Chern–Simons (CS) functional [9,10] in the low energy effective theory variables [6,7], and cancels the anomalies on the boundaries.

With the chiral quarks attached to the boundaries, ψ_L at $x^5 = 0$, and ψ_R at $x^5 = R$, respectively, a "constituent quark mass term" is introduced of the form $m\bar{\psi}_L W \psi_R + h.c.$. Here, W is the Wilson line that spans the gap between the boundary branes, and represents the dynamical chiral condensate of the theory. The Wilson line is identified with a chiral field of mesons:

$$W(x^\mu) = \exp\left(i\int_0^R dx^5 A_5(x^\mu, x^5)\right) \equiv \exp\left(\frac{ia(x^\mu)}{f}\right). \tag{1}$$

This is the rationale for requiring an A_5 zero mode, since we need that the pseudoscalar chiral meson a be physical, and not be eaten by a KK-mode. The chiral symmetry breaking scale is specified by f, the "pion decay constant"; in any imitation of QCD chiral dynamics by an extra dimension, chiral symmetry breaking is generically related to the compactification scale, *i.e.*, $f \sim 1/R$.

The quark anomalies [8] on the boundary branes must be canceled by the anomalies that arise on the boundaries from a bulk-filling Chern–Simons functional. The anomaly cancelation condition determines the coefficient of the Chern–Simons functional [10]. We integrate out the quarks, taking the limit of large m. In a special gauge, an effective "Fujikawa" action arises out of the quark Dirac determinant [11], which only involves the gauge fields on the boundary, and amounts to minus the Bardeen counterterm [3,5,8].

When the Chern–Simons term and the boundary term are added together, they yield a total effective action, S^*, all of whose anomalies cancel. Through a suitable reverse gauge transformation, we finally arrive at the purely bosonic anomaly-free action, consisting of a surface term and bulk term,

$$S^* = \Gamma_{WZW}(a(x^\mu), A_L(x^\mu), A_R(x^\mu)) + S_{CS}(A_A(x^\mu, x^5)). \tag{2}$$