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Effective theory for the Goldstone field in the BCS–BEC crossover at $T = 0$

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We perform a detailed study of the effective Lagrangian for the Goldstone mode of a superfluid Fermi gas at zero temperature in the whole BCS–BEC crossover. By using a derivative expansion of the response functions, we derive the most general form of this Lagrangian at the next to leading order in the momentum expansion in terms of four coefficient functions. This involves the elimination of all the higher order time derivatives by careful use of the leading order field equations. In the infinite scattering length limit where conformal invariance is realized, we show that the effective Lagrangian must contain an unnoticed invariant combination of higher spatial gradients of the Goldstone mode, while explicit couplings to spatial gradients of the trapping potential are absent. Across the whole crossover, we determine all the coefficient functions at the one-loop level, taking into account the dependence of the gap parameter on the chemical potential in the mean-field approximation. These results are analytically expressed in terms of elliptic integrals of the first and second kind. We discuss the form of these coefficients in the extreme BCS and BEC regimes and around the unitary limit, and compare with recent work by other authors.

1. Introduction

The last few years have witnessed a renewed interest in the physics of the BCS–BEC crossover [1–3], partly motivated by the availability of tunable interactions in the realm of interacting Fermi

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gases [4]. Recent experimental work [5–9] has shown evidence for condensation of fermionic atom pairs, suggesting the formation of a fermionic superfluid. From the theoretical point of view, the qualitative description of the BCS–BEC crossover has been based on the mean-field theory of Leggett [2] and its extension to finite temperature by Nozières and Schmitt-Rink [10] and Sá de Melo, Randeria and Engelbrecht [11]. In this description of the superconducting system, the effective action in terms of a complex order parameter which couples to the pairing field plays a central role. Some recent developments [12–15] in the crossover problem beyond mean-field theory have improved our understanding of the equilibrium state at a quantitative level. In particular, Diener et al. [15], by computing the complete quadratic part of the effective action, have obtained the correction to the mean-field result which arises from the integration of the Gaussian fluctuations, finding excellent agreement with calculations based on quantum Monte Carlo techniques in the unitary limit where the scattering length $a \to \infty$.

The effective action can also used, in principle, to derive an effective Lagrangian which captures the low-energy behavior of the system in terms of the Goldstone mode phase of the order parameter. At zero temperature, one expects that an expansion of the effective Lagrangian in derivatives of the Goldstone field can be used in order to study the low-energy behavior of the system. The leading order (LO) in this expansion was evaluated by Greiter et al. [16] and by Aitchison et al. [17] some years ago, and since then, there have been various microscopic derivations [18–20,22,21] of effective models, some of which [19,20] have included more or less explicitly some derivative corrections.

In a recent work, Son and Wingate [23] have systematically studied the form of the effective Lagrangian in the unitary limit at the next-to-leading order (NLO) in the derivative expansion, when the effective theory is formulated in terms of the Goldstone mode coupled to external gauge and gravitational fields. At unitarity, it turns out that, besides general coordinate and gauge invariance, the theory exhibits conformal invariance [24], which puts constraints on the form of the NLO Lagrangian and restricts to two the number of independent NLO parameters.

In this paper, we extend these studies. We evaluate the effective parameters of the NLO Lagrangian in the unitary limit at the mean-field level, and also obtain the most general form of this Lagrangian away from this limit, where the symmetry under conformal transformations is not realized. By computing all the necessary functions at the one-loop level in terms of elliptic integrals, we have obtained the simplest approximation to the low-energy effective theory for the whole crossover region at zero temperature.

We find that, at unitarity, the effective Lagrangian is specified by two constants, but its actual form differs from that given in [23], and includes a new contribution $(\partial_i\partial_j\theta)^2$ of higher spatial derivatives of the Goldstone mode, while the NLO contribution of the external trapping potential proportional to $\nabla^2 V_{\text{ext}}$ is absent. We show that these features are a necessary consequence of the conformal invariance of the NLO field equations. As an application, we derive the energy density functional in the unitary limit, and compare it with the computation of Rupak and Schäffer [25], which is based on an epsilon expansion around $d = 4 - \varepsilon$ spatial dimensions and is, to our knowledge, the only one in the literature. Altough the coefficients computed by these two methods show discrepancies of the order of 30% in general, we find a surprisingly good agreement for the coefficient of the quantum pressure.

For the whole crossover region, we obtain the NLO Lagrangian in terms of four functions which are given in closed form in terms of elliptic integrals. The BCS and BEC limits as well as the near unitarity limits of the NLO Lagrangian are worked out in detail. In the BEC limit, we recover the known features of the hydrodynamic description of superfluidity at zero temperature.

The plan of this paper is as follows. In Section 2, we formulate the problem in the framework of linear response theory and derive a linearized equation for the Goldstone field in terms of derivatives of response functions. In Section 3, we show how to construct a Lagrangian, including second order time derivatives of θ, by considering all the available Galilean invariants consistent with required general properties. Then, we present a careful procedure of reduction and show how to use the LO field equation in order to eliminate undesired higher order time derivatives without changing its perturbative contents. We also argue the need to compute two three-point functions in order to determine all the coefficient functions in the effective Lagrangian. In Section 4, we present an analytical expression for the thermodynamical potential at the one-loop level, and the analytical expressions of the NLO coefficients in the two (BCS and BEC) limits and near unitarity. In Section 5, we compute the energy