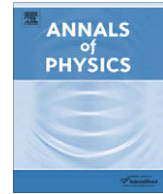




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The geometry of Schrödinger symmetry in non-relativistic CFT

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ABSTRACT

The non-relativistic conformal “Schrödinger” symmetry of some gravity backgrounds proposed recently in the AdS/CFT context, is explained in the “Bargmann framework”. The formalism incorporates the Equivalence Principle. Newton–Hooke conformal symmetries, which are analogs of those of Schrödinger in the presence of a negative cosmological constant, are discussed in a similar way. Further examples include topologically massive gravity with negative cosmological constant and the Madelung hydrodynamical description.

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1. Introduction

Non-relativistic conformal transformations have initially been discovered as those space-time transformations that permute the solutions of the free Schrödinger equation [1,2]. In $D + 1$ dimensional non-relativistic space-time with position coordinates \mathbf{y} and time y we have, in addition to the (one-parameter centrally extended) Galilean generators, also

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$$\begin{aligned}
 (\mathbf{y}, t) &\rightarrow (\mathbf{y}^*, t^*) = (\alpha \mathbf{y}, \alpha^2 t) \quad \text{dilatation,} \\
 (\mathbf{y}, t) &\rightarrow (\mathbf{y}^*, t^*) = \left(\frac{\mathbf{y}}{1 - \kappa t}, \frac{t}{1 - \kappa t} \right) \quad \text{expansions,}
 \end{aligned} \tag{1}$$

referred to as “non-relativistic conformal transformations”. Added to the Galilean symmetries provides us with the Schrödinger group. Dilatations, expansions and time translations span an $\mathfrak{o}(2, 1) \approx \mathfrak{sl}(2, \mathbb{R})$ subalgebra.

These rather mysterious extra symmetries have been identified as the isomorphisms of the structure of non-relativistic space-time [3,4]. For “empty” space, one gets, in particular, the (one-parameter centrally extended) Schrödinger group. Conformal transformations act as symmetries also when an inverse-square potential is introduced [1,5]. In $D = 3$ a Dirac monopole can be included [6,7], and in $D = 2$ one can have instead a magnetic vortex [8]. Full Schrödinger symmetry is restored for a matter field interacting with a Chern-Simons gauge field [9,10]. It can also be present in hydrodynamics [11,12]. See also [13].

Recently, the AdS/CFT correspondence has been extended to non-relativistic field theory [14–17]. The key point is to use the metric

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{r^2} \left[d\mathbf{x}^2 + dr^2 + 2 dt ds - \frac{dt^2}{r^2} \right] = \frac{1}{r^2} g_{\mu\nu} dx^\mu dx^\nu, \tag{2}$$

where \mathbf{x} is an d -dimensional vector and r an additional coordinate. The metric (2) is a $d + 3$ dimensional relativistic space-time, conformally related to the pp-wave defined by $g_{\mu\nu}$ on the same manifold. The interesting feature of the metric (2) is that its *isometries* are the *conformal* transformations of $d + 1$ dimensional non-relativistic space-time, with coordinates (\mathbf{x}, t) .

Below explain the construction and properties of this metric, and illustrate it on some physical examples.

2. Siklos spacetimes

The metric (2) belongs to the class of Siklos spacetimes [18–20], interpreted as exact gravitational waves traveling along AdS ,

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{r^2} \left[d\mathbf{x}^2 + dr^2 + 2 dt ds - F(\mathbf{x}, r, t) dt^2 \right]. \tag{3}$$

$\xi = \partial_s$ is a null Killing vector. The metric (3) can also be presented as $\bar{g}_{\mu\nu} = \bar{g}_{\mu\nu}^{AdS} - r^2 F \xi_\mu \xi_\nu$, generalizing the familiar Kerr–Schild transformation. For $F = 0$, (3) reduces to the anti-de Sitter metric.

The Einstein tensor of (3) satisfies

$$\begin{aligned}
 \bar{G}_{\mu\nu} + \Lambda \bar{g}_{\mu\nu} &= \rho \xi_\mu \xi_\nu, \\
 \rho &= \frac{r^4}{2} \left(\partial_r^2 F - \frac{d+1}{r} \partial_r F + \Delta_{\mathbf{x}} F \right),
 \end{aligned} \tag{4}$$

where $\Lambda = -(d+1)(d+2)/2$. Hence, these spacetimes are solutions of gravity with a negative cosmological constant, coupled to light-like fluid. The only non-vanishing component of the Einstein–de Sitter tensor is $\bar{G}_{tt} + \Lambda \bar{g}_{tt}$.

The RHS of (4) is traceless, since it is the energy–momentum tensor of some relativistic fluid made of massless particles. When F satisfies the Siklos equation [19]

$$\partial_r^2 F - \frac{d+1}{r} \partial_r F + \Delta_{\mathbf{x}} F = 0, \tag{5}$$

then $\rho = 0$ and (3) is the AdS_{d+3} metric.

The effect of a conformal redefinition of the metric, $g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ has been studied by Brinkmann [21]. Applied to our case, we see that the Einstein equation of the pp-wave $g_{\mu\nu}$ in (1) goes over, for $\Omega = r^{-1}$, to that with negative cosmological constant, appropriate for $\bar{g}_{\mu\nu}$.

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