Available online at www.sciencedirect.com
ScienceDirect
Annals of Physics 323 (2008) 1989–2017
ANNALS of PHYSICS
www.elsevier.com/locate/aop

Causal signal transmission by quantum fields. II: Quantum-statistical response of interacting bosons

L.I. Plimak [a,*], S. Stenholm [a,b,c]

[a] Abteilung Quantenphysik, Universität Ulm, D-89069 Ulm, Germany
[b] Physics Department, Royal Institute of Technology, KTH, Stockholm, Sweden
[c] Laboratory of Computational Engineering, HUT, Espoo, Finland

Received 8 October 2007; accepted 16 November 2007
Available online 11 January 2008

Abstract

We analyse nonperturbatively signal transmission patterns in Green's functions of interacting quantum fields. Quantum field theory is reformulated in terms of the nonlinear quantum-statistical response of the field. This formulation applies equally to interacting relativistic fields and nonrelativistic models. Of crucial importance is that all causality properties to be expected of a response formulation indeed hold. Being by construction equivalent to Schwinger's closed-time-loop formalism, this formulation is also shown to be related naturally to both Kubo's linear response and Glauber's macroscopic photodetection theories, being a unification of the two with generalisation to the nonlinear quantum-statistical response problem. In this paper we introduce response formulation of bosons; response reformulation of fermions will be subject of a separate paper.

PACS: 03.70.+k; 05.30.−d; 05.70.Ln

Keywords: Quantum-statistical response problem; Quantum field theory; Phase-space methods

* Corresponding author.
E-mail address: lev.plimak@uni-ulm.de (L.I. Plimak).

0003-4916/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.aop.2007.11.014

1. Introduction

This paper continues the investigation of response properties of quantum systems started in Ref. [1]. In [1] we introduced a *response formulation* of the harmonic oscillator and extended it to noninteracting bosonic fields. Here we show that response formulation may be further extended to arbitrary interacting bosonic fields. Response reformulation of fermions will be subject of a separate paper.

For noninteracting bosons [1], response formulation means description of the quantum system in terms of quantum averages of the *normally-ordered products* [2–5] of field operators defined in the presence of external sources. In [1], we proved that this description is equivalent to the standard quantum-field-theoretical description of the same system within Schwinger's renown closed-time-loop formalism [6]. The response formulation and the Schwinger formalism are coupled by a one-to-one *response substitution* in the corresponding characteristic functionals.

It would seem that the obvious way of generalising this result to interacting fields is replacing the normal ordering of free-field operators by the *time-normal* ordering of Heisenberg operators as introduced by Glauber and Kelly and Kleiner [4,5]. However, we quickly discover that this leads to loss of the key property of the response formulation: its equivalence to the standard Green-function approach. Without an amendment to the concept of time-normal ordering, response substitution for interacting bosons does not exist. This makes the whole exercise pointless: recall that our ultimate goal is extending the phase-space techniques to relativistic problems. We are therefore forced to choose a different approach. We take the response substitution found in [1] for noninteracting bosons, postulate it for interacting bosons, and consider the consequences. This implies introducing a new definition replacing the familiar time-normal operator ordering. However, we also show that within the optical paradigm (technically, within the approximation of slowly varying amplitudes) this definition coincides with the definition by Glauber and Kelly and Kleiner. Interpretation of any of the quantum-optical experiments needs not be reconsidered.

Except being a preparatory work for the phase-space approach to relativistic quantum fields, some results of this paper appear to have significance of their own. First and foremost, we demonstrate that all quantum properties of interacting systems may be interpreted in terms of response and self-radiation. As was explained in paper [1], one interpretation of our results is proving the equivalence between Schwinger's closed-time-loop formalism [6] on the one hand, and a certain generalisation of Kubo's and Glauber's approaches combined on the other. For more details we refer the reader to the introduction of Ref. [1]. All points made there apply not just to the harmonic oscillator, but also to any interacting bosonic quantum system.

Perhaps the most interesting result of this paper is a fundamental link between response and noncommutivity of operators. Indication of this connection may be seen already in Kubo's famous formula for the linear response function [7], where the latter is expressed by the average of the two-time commutator. This feature is shown to hold for the full nonlinear quantum-statistical response of interacting systems. The assumption that operators commute cancels the dependence of the system properties on external sources turning the system into a "pre-assigned quantum source."

Another interesting result, which plays only a technical role in our analyses but seems to be important on its own, is that all response properties of a system are contained in the