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Weighted power counting and Lorentz violating gauge theories. I: General properties

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ABSTRACT

We construct local, unitary gauge theories that violate Lorentz symmetry explicitly at high energies and are renormalizable by weighted power counting. They contain higher space derivatives, which improve the behavior of propagators at large momenta, but no higher time derivatives. We show that the regularity of the gauge-field propagator privileges a particular spacetime breaking, the one into space and time. We then concentrate on the simplest class of models, study four dimensional examples and discuss a number of issues that arise in our approach, such as the low-energy recovery of Lorentz invariance.

1. Introduction

Lorentz symmetry has been verified in many experiments with great precision [1]. However, different types of arguments have lead some authors to suggest that it could be violated at very high energies [2–4]. This possibility has raised a considerable interest, because, if true, it would substantially affect our understanding of Nature. The Lorentz violating extension of the Standard Model [3] contains a large amount of new parameters. Bounds on many of them, particularly those belonging to the power-counting renormalizable subsector, are available. Their updated values are reported in Ref. [5].

In quantum field theory, the classification of local, unitarity, polynomial and renormalizable models changes dramatically if we do not assume that Lorentz invariance is exact at arbitrarily high energies [6,7]. In that case, higher space derivatives are allowed and can improve the behavior of propagators at large momenta. A number of theories that are not renormalizable by ordinary power counting become renormalizable in the framework of a "weighted power counting" [6], which assigns

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different weights to space and time, and ensures that no term containing higher time derivatives is generated by renormalization, in agreement with unitarity. Having studied scalar and fermion theories in Refs. [6,7], here we begin the study of gauge theories, focusing on the simplest class of models. The investigation is completed in a second paper [8], to which we refer as paper II, which contains the classification of renormalizable gauge theories.

The theories we are interested in must be local and polynomial, free of infrared divergences in Feynman diagrams at non-exceptional external momenta, and renormalizable by weighted power counting. We find that in the presence of gauge interactions the set of renormalizable theories is more restricted than in the scalar-fermion framework. Due to the particular structure of the gauge-field propagator, Feynman diagrams are plagued with certain spurious subdivergences. We are able to prove that they cancel out when spacetime is broken into space and time, and certain other restrictions are fulfilled.

A more delicate physical issue is the low-energy recovery of Lorentz symmetry. Once Lorentz symmetry is violated at high energies, its low-energy recovery is not guaranteed, because renormalization makes the low-energy parameters run independently. One possibility is that the Lorentz invariant surface is RG stable [9], otherwise a suitable fine-tuning must be advocated.

In other domains of physics, such as the theory of critical phenomena, where Lorentz symmetry is not a fundamental requirement, certain scalar models of the types classified in Ref. [6] have already been studied [10] and have physical applications.

The paper is organized as follows. In Section 2 we review the weighted power counting for scalar-fermion theories. In Section 3 we extend it to Lorentz violating gauge theories and define the class of models we focus on in this paper. We study the conditions for renormalizability, absence of infrared divergences in Feynman diagrams and regularity of the propagator. In Section 4 we prove that the theories are renormalizable to all orders, using the Batalin–Vilkovisky formalism. In Section 5 we study four dimensional examples and the low-energy recovery of Lorentz invariance. In Section 6 we discuss strictly renormalizable and weighted scale invariant theories. In Section 7 we study the Proca Lorentz violating theories, and prove that they are not renormalizable. Section 8 contains our conclusions. In Appendix A we classify the quadratic terms of the gauge-field lagrangian and in Appendix B we derive sufficient conditions for the absence of spurious subdivergences.

2. Weighted power counting

In this section we briefly review the weighted power counting criterion of Refs. [6,7]. The simplest framework to study Lorentz violations is to assume that the d-dimensional spacetime manifold $M = \mathbb{R}^d$ is split into the product $\widehat{M} \times \overline{M}$ of two submanifolds, a $\hat{d}$-dimensional submanifold $\widehat{M} = \mathbb{R}^{\hat{d}}$, containing time and possibly some space coordinates, and a $\bar{d}$-dimensional space submanifold $\overline{M} = \mathbb{R}^{\bar{d}}$, so that the d-dimensional Lorentz group $O(1, d-1)$ is broken to a residual Lorentz group $O(1, \hat{d}-1) \times O(\bar{d})$. In this paper we study renormalization in this simplified framework. The generalization to the most general breaking is done in paper II.

The partial derivative ∂ is decomposed as $\left(\hat{\partial}, \bar{\partial}\right)$, where $\hat{\partial}$ and $\bar{\partial}$ act on the subspaces $\widehat{M}$ and $\overline{M}$, respectively. Coordinates, momenta and spacetime indices are decomposed similarly. Consider a free scalar theory with (Euclidean) lagrangian

$$\mathcal{L}_{\text{free}} = \frac{1}{2}\left(\hat{\partial}\varphi\right)^2 + \frac{1}{2\Lambda_L^{2n-2}}\left(\bar{\partial}^n\varphi\right)^2, \tag{2.1}$$

where Λ_L is an energy scale and n is an integer >1. Up to total derivatives it is not necessary to specify how the $\bar{\partial}$'s are contracted among themselves. The coefficient of $\left(\bar{\partial}^n\varphi\right)^2$ must be positive to have a positive energy in the Minkowskian framework. The theory (2.1) is invariant under the weighted rescaling

$$\hat{x} \to \hat{x}\,e^{-\Omega}, \qquad \bar{x} \to \bar{x}\,e^{-\Omega/n}, \qquad \varphi \to \varphi\,e^{\Omega(\not{d}/2-1)}, \tag{2.2}$$

where $\not{d} = \hat{d} + \bar{d}/n$ is the "weighted dimension". Note that Λ_L is not rescaled.