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Quantum field theory with an interaction on the boundary

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Abstract

We consider quantum theory of fields ϕ defined on a D dimensional manifold (bulk) with an interaction $V(\phi)$ concentrated on a $d < D$ dimensional surface (brane). Such a quantum field theory can be less singular than the one in d dimensions with an interaction $V(\phi)$. It is shown that scaling properties of fields on the brane are different from the ones in the bulk. We discuss as an example fields on de Sitter space.

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1. Introduction

Models with an interaction concentrated on a $d < D$ dimensional submanifold (brane) of a D dimensional manifold (bulk) are interesting for high energy physics as well as for statistical physics. In the first case we consider the visible universe as a submanifold (a brane [1,2]) of a higher dimensional space. Field theoretic models with an interaction on the boundary come also from string theory [3]. In Ref. [3] the 11-dimensional gravity is interacting with 10-dimensional gauge fields living on the boundary. In statistical

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physics we may consider materials with a boundary and an interaction of some constituents placed on the boundary [4–7]. It is an experimental fact [8] that correlation functions of field variables depending on the boundary points have critical exponents different from the bulk correlation functions.

In this paper, we discuss field theoretic models with an interaction on the brane. We concentrate on the scalar field but some methods and results can be generalized to gravitational and gauge field interactions. We begin with the free propagator. We admit any boundary condition preserving the symmetries of the free Lagrangian. We show that a differential operator which is singular close to the brane has the Green function which is more regular on the brane than the one for operators with constant coefficients. Subsequently, we discuss models with an interaction concentrated on the brane. The functional measure is defined [9] by its covariance (the Green function) and its mean. The mean breaks symmetries of the classical action. We average over mean values in order to preserve the symmetries. As a consequence of the more regular behaviour of the Green function the model with an interaction concentrated on the brane has milder ultraviolet divergencies. We give examples of nonrenormalizable theories in the bulk which become superrenormalizable when restricted to the boundary. We work mainly with the imaginary time version of quantum field theory. In the last section we discuss the scattering theory. The free particle is treated as a packet of waves on a curved manifold. We calculate the scattering matrix of such particles resulting from the $V(\phi)$ interaction concentrated either on the boundary or at the time $z = 0$ (a kick at a fixed moment).

2. Green functions on a boundary

We consider a $D = d + m$ dimensional Riemannian manifold of the warped form $\mathcal{M}_g = \mathcal{M}_m \times_g R^d$ [10] with the boundary R^d whose metric close to the boundary takes the form

$$ds^2 = G_{AB}(X)\,dX^A\,dX^B = g_{\mu\nu}(y)\,dx^\mu\,dx^\nu + g_{jk}(y)\,dy^j\,dy^k, \tag{1}$$

where $X = (y,x)$ are local coordinates on $\mathcal{M}_g$, g_{jk} is the Riemannian metric induced on $\mathcal{M}_m$ and $g_{\mu\nu} : \mathcal{M}_m \to R^{d^2}$ is a positive definite $d \times d$ matrix function defined on $\mathcal{M}_m$. The action for the free field ϕ reads

$$W_0 = \int dX\sqrt{G}G^{AB}\partial_A\phi\partial_B\phi. \tag{2}$$

The free (Euclidean) quantum field can be defined as the one whose propagator is determined by the Green function

$$-\mathcal{A}\mathcal{G} \equiv \partial_A G^{AB}\sqrt{G}\partial_B\mathcal{G} = \delta, \tag{3}$$

where $G = \det G_{AB}$. In the metric (1) Eq. (3) can be expressed as

$$\left(g^{\mu\nu}(y)\sqrt{G}(y)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} + \frac{\partial}{\partial y^j}g^{jk}(y)\sqrt{G}(y)\frac{\partial}{\partial y^k}\right)\mathcal{G} = \delta. \tag{4}$$

The solution of Eq. (3) is not unique. If $\mathcal{G}'$ is another solution of Eq. (3) then $\mathcal{G}' = \mathcal{G} + \mathcal{R}$ where $\mathcal{R}$ is a solution of the equation

$$\mathcal{A}\mathcal{R} = 0. \tag{5}$$