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Orbital currents and charge density waves in a generalized Hubbard ladder

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Abstract

We study a generalized Hubbard model on the two-leg ladder at zero temperature, focusing on a parameter region with staggered flux (SF)/d-density wave (DDW) order. To guide our numerical calculations, we first investigate the location of a SF/DDW phase in the phase diagram of the half-filled weakly interacting ladder using a perturbative renormalization group (RG) and bosonization approach. For hole doping δ away from half-filling, finite-system density-matrix renormalization-group (DMRG) calculations are used to study ladders with up to 200 rungs for intermediate-strength interactions. In the doped SF/DDW phase, the staggered rung current and the rung electron density both show periodic spatial oscillations, with characteristic wavelengths $2/\delta$ and $1/\delta$, respectively, corresponding to ordering wavevectors $2k_{\mathrm{F}}$ and $4k_{\mathrm{F}}$ for the currents and densities, where $2k_{\mathrm{F}} = \pi(1-\delta)$. The density minima are located at the anti-phase domain walls of the staggered current. For sufficiently large dopings, SF/DDW order is suppressed. The rung density modulation also exists in neighboring phases where currents decay exponentially. We show that most of the DMRG results can be qualitatively understood from weak-coupling RG/bosonization arguments. However, while these arguments seem to suggest a crossover from non-decaying correlations to power-law decay at a length scale of order $1/\delta$, the DMRG results are consistent with a true long-range order scenario for the currents and densities.

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1. Introduction

The possibility of finding new phases of matter is a major motivation behind the study of materials of strongly correlated electrons. A phase with staggered orbital currents which was first considered theoretically in 1968 [1] and then rediscovered two decades later [2–6] has lately been the subject of a revival of interest, mainly due to recent proposals [7,8] that the pseudogap region [9] in the phase diagram of the cuprate high-temperature superconductors may be characterized by this kind of order, either long-ranged (i.e., a true broken-symmetry state) [7], or fluctuating [8]. This orbital current phase is known variously as the orbital antiferromagnet, staggered flux (SF) or d-density wave [10] (DDW) phase; in this paper we will refer to it as the SF/DDW phase. The long-range ordered version of this phase breaks the rotational and translational symmetries of the underlying Hamiltonian, in addition to time reversal symmetry. The fundamental experimental signature of the long-range-ordered SF/DDW scenario with ordering wavevector (π, π) is an elastic Bragg peak at that wavevector in neutron scattering [6,11a,b]. The results of some recent neutron scattering experiments on underdoped $YBa_2Cu_3O_{6+x}$ have been argued to be consistent with this scenario [12a,b,11a,b,13]. A different circulating-current broken-symmetry state with a current pattern that does not break translational symmetry has also been proposed for the pseudogap phase in the cuprates [14a–c].

It is important to acquire an understanding of what kinds of microscopic models can give rise to a SF/DDW ground state. For models of interacting fermions in two spatial dimensions this issue has been addressed by many authors [2–6,16,17,15,18–25] using a variety of methods. While the two-dimensional case is the most relevant one for the cuprates, the behavior of strongly correlated electrons in two dimensions is still a subject that is marred by great controversy. In a few special model cases the fermion sign problem is absent and thus quantum Monte Carlo simulations can be done reliably, leading among other things to interesting findings with regard to SF/DDW order [25] and also more exotic current-carrying states [26]. However, in the vast majority of cases, and certainly for the models and parameter values that are expected to be most relevant for real materials, the available analytical and numerical methods are only approximate, and their reliability is difficult to gauge.

In contrast, the fact that many powerful methods exist for one spatial dimension has enabled much solid knowledge to be established about strongly correlated one-dimensional systems, and it is hoped that some of this may also be relevant for the behavior of correlated electrons in two dimensions. In this regard, two-leg Hubbard and t–J ladders have attracted much interest, as it has been found that these models have a spin gap and that upon doping away from half-filling the dominant correlations are d-wave-like superconducting correlations [27a,b], two features which are reminiscent of the pseudogap and the $d_{x^2-y^2}$ symmetry of the superconducting state in the cuprates, respectively. Ladder systems are also interesting in their own right, not least due to experimental realizations of such materials. The most well-known example, $Sr_{14-x}Ca_xCu_{24}O_{41}$, contains two-leg ladder substructures, and has been found to have a spin gap, and to become superconducting upon doping at high pressure [28,27a,b].

The possibility of having SF/DDW order in two-leg ladders has also received much attention recently [29–41]. The two-leg ladder is the simplest geometry that can support a SF/DDW phase, with currents flowing around the elementary square plaquettes. In this paper, we present an extensive study of SF/DDW order in generalized Hubbard ladders,