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Effective field theories for superconducting systems with multiple Fermi surfaces

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ABSTRACT

In this work we investigate the description of superconducting systems with multiple Fermi surfaces. For the case of one Fermi surface we re-obtain the result that the superconductor is more precisely described as a topological state of matter. Studying the case of more than one Fermi surface, we obtain the effective theory describing a time reversal symmetric topological superconductor. These results are obtained by employing a general procedure to construct effective low energy actions describing states of electromagnetic systems interacting with charges and defects. The procedure consists in taking into account the proliferation or dilution of these charges and defects and its consequences for the low energy description of the electromagnetic response of the system. We find that the main ingredient entering the low energy characterization of the system with more than one Fermi surface is a non-conservation of the canonical supercurrent triggered by particular vortex configurations.

1. Introduction

In this work we investigate the low energy description of electromagnetic systems as a function of configurations of charged sources and defects. We specifically address the case when such systems are superconducting and may display multiple Fermi surfaces.

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This present work was mainly motivated by the illuminating study developed in [1], where the authors centered the analysis of a superconductor state on the proper identification of its low energy degrees of freedom. This leads to the conclusion that the effective low energy theory of a superconductor, usually described by Ginzburg–Landau theory, is more appropriately described by a topological *BF* theory encoding the topological interactions of vortices and charges in the system. This strategy led us to consider the use of a procedure, known as the Julia–Toulouse approach (JTA) [2], which naturally accommodates such a focus on the behavior of the degrees of freedom. The JTA has the aim of constructing an effective field theory for gauge fields from semi-classical considerations about the collective dynamics of charged particles and defects in the system. The procedure is thoroughly reviewed in [3] and has already been employed for the study of superconducting systems in relation to confinement [4], see also [5,6] for related work.

Another motivation for the present work comes from the recent studies of Qi, Witten and Zhang [7]. These authors proposed an effective theory for a time reversal invariant topological superconductor in three spatial dimensions, $(3+1)D$ (for a review of topological materials see [8]). Their construction involves consideration of a time reversal topological insulator in $(4+1)D$ sandwiched between two boundaries, where each boundary sustain a $(3+1)D$ *s*-wave superconductor. This construction leads, upon dimensional reduction, to an effective description of topological superconductor characterized by a topological term describing a coupling between the electromagnetic field and the superconducting phase fluctuation. Such a coupling is mathematically the same as the one between an Abelian gauge field and an axion. One of the main results presented in their work is the realization of the phenomenon of anomaly inflow [9] due to the contribution of so called "chiral vortices".

In this work we provide a construction of the effective field theory of superconductor states that encompasses the observations in [1], thus recovering their findings. Further, we show that this construction can tackle the case of multiple Fermi surfaces as well, and we show that this naturally leads to the results of [7]. This construction thus highlights the fact that the main element characterizing the class of topological superconductors discussed in [7] is the presence of more than one Fermi surface, such that the single Fermi surface superconductor discussed in [1] can be viewed as a special case. Furthermore we clarify the role played by the different vortices in the system providing a precise relation between them and the anomaly of the supercurrent. We observe that an apparent paradox in the results of [7] was reported in [10], where the authors pointed out that Majorana fermions, being uncharged, could not account for the anomaly inflow and proceed to propose a solution to the anomaly imbalance (for a review about the role of Majorana fermions in topological superconductor see [11,12]). In the present work we add to these findings by showing that the non-conservation of the supercurrent is a consequence of the existence of a particular configuration of vortices that do not carry electromagnetic flux. Nevertheless this is just an apparent anomaly restricted to the non-conservation of the canonical supercurrent, since gauge symmetry is maintained throughout the whole procedure and the total electric current in the system is conserved, as it should.

This work is organized as follows. In Section 2 we present the general description of a superconductor state from the point of view of the JTA relying on a parametric condensation or dilution of vortices in the system. This section will provide the main concepts and ingredients for the following sections. In Section 2.1 we present our first main result which is the recovering of the results of [1] under our formalism. In Section 3 we discuss the case of a system displaying two Fermi surfaces. A superconductor is characterized by the dynamics of excitations near a Fermi surface. The key observation here is to allow for charge transfer between Fermi surfaces. This gives rise to a non-trivial topological interaction between different Fermi surfaces and results to be the origin of the topological properties obtained in the effective theory put forward by [7]. In Section 4 we generalize the results for an arbitrary number of Fermi surfaces. In Section 5 we present our conclusions.

2. Effective theory for a superconductor and condensation

Consider the Euclidean action defining a gauge field interacting with a classical source in $4D$

$$S_{em} = \int d^4x \left(\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - iqA_\mu J_\mu \right) \tag{1}$$