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# Dirac equation in 2-dimensional curved spacetime, particle creation, and coupled waveguide arrays

Christian Koke [a], Changsuk Noh [b,*], Dimitris G. Angelakis [c,d]

[a] Institut für theoretische Physik, Philosophenweg 16, D-69120 Heidelberg, Germany

[b] Korea Institute for Advanced Study, 85 Hoegiro, Seoul 130-722, Republic of Korea

[c] Centre for Quantum Technologies, National University of Singapore, 2 Science Drive 3, 117542, Singapore

[d] School of Electronic and Computer Engineering, Technical University of Crete, Chania, Crete, 73100, Greece

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ABSTRACT

When quantum fields are coupled to gravitational fields, spontaneous particle creation may occur similarly to when they are coupled to external electromagnetic fields. A gravitational field can be incorporated as a background spacetime if the back-action of matter on the field can be neglected, resulting in modifications of the Dirac or Klein–Gordon equations for elementary fermions and bosons respectively. The semi-classical description predicts particle creation in many situations, including the expanding-universe scenario, near the event horizon of a black hole (the Hawking effect), and an accelerating observer in flat spacetime (the Unruh effect). In this work, we give a pedagogical introduction to the Dirac equation in a general 2D spacetime and show examples of spinor wave packet dynamics in flat and curved background spacetimes. In particular, we cover the phenomenon of particle creation in a time-dependent metric. Photonic analogs of these effects are then proposed, where classical light propagating in an array of coupled waveguides provides a visualisation of the Dirac spinor propagating in a curved 2D spacetime background. The extent to which such

* Corresponding author.
*E-mail addresses:* christian.koke@stud.uni-heidelberg.de (C. Koke), changsuk@kias.re.kr (C. Noh), dimitris.angelakis@gmail.com (D.G. Angelakis).

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a single-particle description can be said to mimic particle creation is discussed.



## 1. Introduction

Quantum theory of gravity is one of the most sought-after goals in physics. Despite continuous efforts to tackle this important problem, resulting in interesting proposals such as superstring theory and loop quantum gravity, there is still no clear sign of the theory of quantum gravity [1–3]. However, when the back-action of matter on the gravitation field can be neglected, one can write down equations for quantum fields in curved spacetime by extending quantum field theory in Minkowski metric to a general metric [4–7]. This is analogous to treating external fields as c-numbers and predicts interesting new phenomena that are valid in appropriate regimes. Similarly to the prediction of pair creation in external electric fields [8–10], external gravitational fields (as described by a background spacetime) induce particle creation [11–13]. In the latter, particle creation is caused by the change in the vacuum state itself under quite generic conditions.

In this work, we propose a classical optical simulation of particle creation in binary waveguide arrays. There have been many proposals and experimental demonstrations of a plethora of interesting physics in coupled waveguide arrays [18–30]. In particular, an optical simulation of the 1+1 dimensional Dirac equation in binary waveguide arrays has been proposed [14,15] and experimentally demonstrated [16,17]. We show that the setup can be generalised to also simulate the Dirac equation in 2 dimensional curved spacetime.

Particle creation is by definition a multi-particle and the full simulation of the result requires quantum fields as a main ingredient (for example, see [31] for a simulation of Dirac equation in curved spacetime with cold atoms on optical lattices). However, light propagation in a waveguide array is an intrinsically classical phenomenon, so how can one simulate particle creation in a binary waveguide array? The short answer is that we will be looking at the single-particle analog of particle creation. As we will see, the fundamental reason behind particle creation is the difference in the vacuum state, which in turn is captured by different mode-expansions of quantum fields. We can thus concentrate on a single-mode at a time and simulate the effect. In fact, the well-known Klein paradox shows that the single-particle Dirac equation contains subtle hints of multi-particle effects, and the phenomenon of pair production in strong electric fields has been studied within the single particle picture [32]. We study the time evolution of spinor wave packets and demonstrate that an analog of particle creation can be *visualised* in the light evolution in a binary waveguide array. Here, we stress that by using the phenomenon of 'zitterbewegung' (the jittering motion of a Dirac particle), one can bypass the quantitative checks in 'proving' the simulation of particle creation.

This article is organised as follows. In Section 2, we provide a pedagogical introduction to the Dirac equation in curved spacetime, assuming familiarity with the conventional Dirac equation. In Section 3, we specialise to the 1+1 dimensions and provide a few examples of spacetime metrics. Particle creation in curved spacetime is explained, using scalar fields for simplicity, and the single-particle analogs for the Dirac spinors is discussed. Section 4 shows the wave packet evolution both in flat and curved spacetimes, using time-dependent gravitational fields as an example. A single-particle analog of particle creation in a particular case is explicitly demonstrated. Section 5 explains the optical simulation of the Dirac equation in a binary waveguide array and proposes a generalisation to curved spacetimes. We conclude in Section 6.

We have tried to be as pedagogical and self-contained as we could. We have tried to collect and present essential ideas to understand quantum fields in curved spacetime and how it predicts particle creation. It is our hope that the reader will find this helpful in understanding the essential ideas quickly and develop further interesting analogies.