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Observing quantum trajectories: From Mott's problem to quantum Zeno effect and back

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The experimental results of Kocsis et al., Mahler et al. and the proposed experiments of Morley et al. show that it is possible to construct "trajectories" in interference regions in a two-slit interferometer. These results call for a theoretical re-appraisal of the notion of a "quantum trajectory" first introduced by Dirac and in the present paper we re-examine this notion from the Bohm perspective based on Hamiltonian flows. In particular, we examine the short-time propagator and the role that the quantum potential plays in determining the form of these trajectories. These trajectories differ from those produced in a typical particle tracker and the key to this difference lies in the *active* suppression of the quantum potential necessary to produce Mott-type trajectories. We show, using a rigorous mathematical argument, how the active suppression of this potential arises. Finally we discuss in detail how this suppression also accounts for the quantum Zeno effect.

1. Introduction

There has been a revival of interest in the question of whether any meaning can be given to the notion of a particle trajectory in the quantum domain where field theory has already been

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so successful. However, the question originally raised by Kemmer [1] remains, namely, how do we discuss the limiting process through which the particle is identified by the track it leaves in a tracking device like a bubble chamber. To construct a trajectory, we need the notion of a local momentum, a notion that has been assumed to be ruled out by the uncertainty principle. However, the uncertainty principle is about a *simultaneous measurement* of the position and momentum and cannot answer the question as to whether or not the quantum particle actually *has* a simultaneous value of its position and momentum. A further question remains, namely, that if a local momentum exists, how do we measure it?

Wiseman [2] was one of the first to point out that the weak value of the momentum operator is the local momentum or the Bohm momentum in the Bohm approach. (See also Hiley [3] for a wider perspective.) Duck, Stevenson and Sudarshan [4] have shown how weak values can be measured in what are called weak measurements, a result that immediately opens up the possibility of an experimental investigating the trajectories such as those calculated by Philippidis, Dewdney and Hiley [5] in the interference region of a two-slit interferometer. Indeed, the local momentum (essentially Poynting's vector [6]) of the electromagnetic field has already been measured in the experiments of Kocsis et al. [7] and Mahler et al. [8] who used a quantum dot to generate a weak intensity field which was then passed through a two-slit system. Measurements of the local momentum were then used to construct what they called "average photon trajectories". In this sense they are returning to the notion of a "quantum trajectory" first introduced by Dirac [9].

Although these flow lines have some resemblance to the trajectories of Philippidis et al. [5] referred to above, they cannot be compared directly because the latter are calculated using the Schrödinger equation, whereas photons are excitations of the electro-magnetic field. A treatment of the field approach from the Bohmian point of view has been given by Bohm, Hiley and Kaloyerou [10] and by Kaloyerou [11]. Thus it is not clear that photons can be considered to be travelling along trajectories. However Morley, Edmonds and Barker [12] are now carrying out a similar experiment using, instead, argon atoms with an aim to construct trajectories which can be directly compared with the theoretically predicted ones.

In light of this background, we explore the relation between the trajectories determined in a general interference region and the trajectories that are seen in a particle tracker more closely using the Hamiltonian flow method developed by de Gosson [13] which, in turn, clarifies the Bohm approach as discussed in Bohm and Hiley [14]. This latter approach centres on the real part of the Schrödinger equation under polar decomposition of the wave function, which takes the form

$$\frac{\partial S^{\psi}}{\partial t} + \frac{(\nabla S^{\psi})^2}{2m} + V(x) + Q^{\Psi}(x,t) = 0 \tag{1}$$

where $Q^{\Psi}(x,t)$ is the quantum potential defined by

$$Q^{\Psi}(x,t) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R^{\psi}(x,t)}{R^{\psi}(x,t)}.$$

This equation, the quantum Hamilton–Jacobi equation, like its classical counterpart, enables us to calculate an ensemble of trajectories once we are given a solution of the Schrödinger equation for the experimental situation under investigation. Clearly, the key to the different forms of trajectories lies in the appearance of the quantum potential. We will show how this potential arises naturally in the method of Hamiltonian flows and study its behaviour in more detail.

As two of us [15] have already shown for the particular case of a point source, the quantum motion of the particle always reduces to the classical Hamiltonian trajectory for short times. Thus, in this case the non-appearance of the quantum potential will guarantee classical behaviour. In this paper we generalize our theory to the case of arbitrary initial conditions. In this way we provide a rigorous proof of how the Bohmian approach explains this phenomenon without appealing to any wavefunction collapse.

Our investigations show that the key to the appearance of the classical behaviour is the suppression of the quantum potential. Indeed we find that all quantum phenomena arise from the presence of this term in the real part of the Schrödinger equation. Therefore, its suppression will inhibit quantum transitions and this is what is required to explain the quantum Zeno effect.