Annals of Physics 369 (2016) 45–65

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Non relativistic limit of the Landau–Lifshitz equation: A new equation

G. Ares de Parga [a,*], S. Domínguez-Hernández [b], E. Salinas-Hernández [c]

[a] Dpto. de Física, Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, U. P. Adolfo López Mateos, Zacatenco, C.P. 07738, México D.F., Mexico

[b] Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas, Instituto Politécnico Nacional, Av. Miguel Othón de Mendizábal s/n. Col. La Escalera, U. P. Adolfo López Mateos, Zacatenco, Del. Gustavo A. Madero, C.P. 07738, México D.F., Mexico

[c] Escuela Superior de Cómputo, Instituto Politécnico Nacional, Av. Miguel Othón de Mendizábal s/n. Col. La Escalera, C.P. 07738, México D.F., Mexico

ARTICLE INFO

Article history:
Received 27 December 2015
Accepted 24 February 2016
Available online 8 March 2016

Keywords:
Radiation reaction
Landau–Lifshitz equation
Ford equation
Vlasov equation

ABSTRACT

It is shown that Ford equation is not adequate in general to describe the motion of a charged particle including the reaction force in the non relativistic limit. As in General Relativity where a post-Newtonian method is developed in order to describe the gravitational effects at low velocities and small energies, an extra term inherited from Special Relativity must be added to the Ford equation. This is due to that the new term is greater than the reaction force in many physical situations. The Coulombic case is analyzed showing the necessity of including the new term. Comparison with General Relativity results is analyzed. The Vlasov equation to first order in $1/c^2$ is proposed for the constant electric and magnetic fields.

1. Introduction

The reaction force has been analyzed by many authors from the 19th century. The first attempts were done by Lorentz, Abraham [1] and Planck [2–4] in order to deduce an expression for the self-force in the non-relativistic case. In 1938, Dirac [5] proposed a relativistic equation which coincides with the

* Corresponding author.
E-mail address: gadpau@hotmail.com (G. Ares de Parga).

http://dx.doi.org/10.1016/j.aop.2016.02.012
0003-4916/

Abraham–Lorentz equation in the non-relativistic limit. Due to the physical inconveniences appearing in the solutions of the Lorentz–Dirac equation as the runaway solutions and the preaccelerations, Eliezer [6] deduced a second order differential motion equation for a point charged particle which was apparently free of such difficulties. The idea consists of substituting in the expression of the radiation term the hyperacceleration by the external force. Along the years, other proposals appeared in the literature contributing to the discussion of which is the correct equation of motion for charged particle, particularly the action at a distance theory played an important role as for example the complete absorber assumption of Wheeler and Feynman [7,8] which permits to deduce the Lorentz–Dirac equation. Caldirola [9] and Mo and Papas [10] contributed with interesting new approaches. Although, Yaghjian [11] objected the unphysical solutions of the Lorentz–Dirac equation by giving a finite structure to the charges which is compatible with General Relativity since a point charge will represent a black hole [12], nowadays due to the mathematical approach done by Spohn [13,14] and physical aspects exposed by Rohrlich [15–17], the Landau–Lifshitz equation [18] is considered by many authors as the best tool in order to describe the motion of a charged point particle. Moreover, in General Relativity, considering the point characteristic of a charge as a limit which permit to avoid the black hole behavior of a point particle, a Landau–Lifshitz equation-like [19] has been proposed supporting the validity of the Landau–Lifshitz equation. Recently, Ford and O´Connell [20,21] by using quantum arguments and Langevin equation, deduced an equation of motion for non relativistic case known as the Ford equation which can be physically generalized to Special Relativity obtaining the Eliezer equation [22]. Even if Landau–Lifshitz equation and the Eliezer equation are mathematically different, within the Shen zone [23] where a classical trajectory can be defined for a charged particle and quantum effects may be neglected, the solutions for both equations are similar and the differences cannot be detected [24–26]. Same situations occur when the non relativistic Landau–Lifshitz equation is compared with Ford equation [24]. However, recently Hammond [27,28] has proposed a new equation which solves a paradox with respect to the absence of reaction force predicted by all the cited equations when a constant electric field is applied to a charged particle [29]. Nevertheless, this apparent paradox is denied by other authors following the idea that the radiation exits at the infinity; the energy radiated is taken from the attached fields and consequently the absence of the reaction force is explained [17,30–32].

Finally, most authors consider that the Eliezer equation and the Landau–Lifshitz equation represent the most appropriate equations to describe the relativistic motion of a charged particle and the Ford equation and the non-relativistic Landau–Lifshitz equation are best for the non relativistic case [33,34]. However, if we analyze the Ford equation and the non-relativistic Landau–Lifshitz equation, an extra term inherited from Special Relativity must be added for each different applied force. Indeed, as in General Relativity in order to measure the effects at low energies and long times a post-Newtonian method was developed, in Special Relativity some terms must be kept for low energies and long times which are bigger than the reaction force in its non relativistic version. As a matter of fact, when post-Newtonian method is used to calculate the precession of a charged massive particle due to the interaction with a massive charged particle a series of terms contribute to it [35]. In particular, a term which can be deduced by just using Special Relativity, can be seen as an extra force in Newtonian physics and it results to be bigger than the reaction force in the non relativistic case. Moreover, similar effects appear when the tail term is evaluated by using a post-Newtonian method [12,36] and must be compared with our new term and the reaction force.

The purpose of the article consists of showing that in some cases an extra term must be added to the Ford equation because the reaction term ($\tau_q d\overrightarrow{F}/dt$) is smaller than some terms which come from the equation of motion without reaction force expanded to first order in $1/c^2$. That is an effective force must substitute the regular force in the equation of motion; that is: the equation of motion must be written as

$$m\overrightarrow{a} = \overrightarrow{F}_{eff} + \tau_q \frac{d\overrightarrow{F}}{dt} + \Theta(1/c^4), \tag{1}$$

being τ_q the characteristic time of the charge. This can be applied to quasi-relativistic plasma systems where the temperature belongs to the frontier between Classical Mechanics and Relativistic