Annals of Physics 362 (2015) 36–56

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Wave packet dynamics for a non-linear Schrödinger equation describing continuous position measurements

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HIGHLIGHTS

- Time-dependent solutions for non-linear Schrödinger equation.
- Stability of stationary solutions for non-linear Schrödinger equation.
- Bohmian interpretation and position quantum measurement.

ARTICLE INFO

Article history:
Received 10 February 2015
Accepted 17 July 2015
Available online 29 July 2015

Keywords:
Quantum measurement
Nonlinear Schrödinger equation
Nonlinear physics

ABSTRACT

We investigate time-dependent solutions for a non-linear Schrödinger equation recently proposed by Nassar and Miret-Artés (NM) to describe the continuous measurement of the position of a quantum particle (Nassar, 2013; Nassar and Miret-Artés, 2013). Here we extend these previous studies in two different directions. On the one hand, we incorporate a potential energy term in the NM equation and explore the corresponding wave packet dynamics, while in the previous works the analysis was restricted to the free-particle case. On the other hand, we investigate time-dependent solutions while previous studies focused on a stationary one. We obtain exact wave packet solutions for linear and quadratic

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http://dx.doi.org/10.1016/j.aop.2015.07.019

potentials, and approximate solutions for the Morse potential. The free-particle case is also revisited from a time-dependent point of view. Our analysis of time-dependent solutions allows us to determine the stability properties of the stationary solution considered in Nassar (2013), Nassar and Miret-Artés (2013). On the basis of these results we reconsider the Bohmian approach to the NM equation, taking into account the fact that the evolution equation for the probability density $\rho = |\psi|^2$ is not a continuity equation. We show that the effect of the source term appearing in the evolution equation for ρ has to be explicitly taken into account when interpreting the NM equation from a Bohmian point of view.

1. Introduction

Non-linear Schrödinger equations provide useful tools for modelling diverse physical phenomena and constitute an interesting arena for the analysis of fundamental issues in physics [1–8]. They are also related to other non-linear equations of mathematical physics, such as the non-linear Fokker–Planck equations [9–12]. Indeed, research on non-linear Schrödinger equations constitutes an important component of the more general line of enquiry concerning the exploration of non-linear extensions or modifications of some of the equations governing fundamental laws of physics (see for instance [13–15] and references therein). A non-linear Schrödinger equation was recently proposed by Nassar and Miret-Artés (NM) as an effective description of the evolution of a quantum particle subjected to a continuous position measurement process [1,2].

The measurement of the position of quantum particles plays an essential role among the set of all possible physical measurements. Actually most physical measurements, if not even all of them, can be reduced to the measurement of the position of particles [16,17]. It is due to this that position observables are of fundamental importance in numerous approaches to the quantum measurement problem and related aspects of the foundations of quantum mechanics. Among the interesting proposals that focus on particles' position and which deal with fundamental aspects of quantum mechanics, are the de Broglie–Bohm pilot wave approach to quantum mechanics [18–20], the aforementioned non-linear extension of the Schrödinger equation describing the continuous measurement of the position of a particle [1,2], the Ghirardi–Rimini–Weber model of wave function collapse [21], the entropic-dynamics approach to quantum evolution [22], and the Fisher information-based derivation of the fundamental Lagrangians which leads to relativistic wave equations [23]. Last but not least, an intriguing new perspective on the preponderance of the position observable in our perception of Nature has been recently advanced by Tegmark [24], suggesting possible connections between this issue and the physical basis of consciousness.

In the present contribution we revisit the abovementioned NM approach to continuous position measurement processes, investigating time-dependent solutions of the associated non-linear Schrödinger equation. NM considered the measurement of the position of a free particle, that is, "free" except from the effects arising from the measurement itself, which are modelled by a non-linear term in the Schrödinger equation [1,2]. Here we consider the measurement of the position of a particle moving under the effect of a potential $V(x)$. Our present study goes beyond previous works also because we consider explicitly time-dependent solutions to the NM equation, while previous studies focused on a stationary solution. We obtain exact solutions for linear and quadratic potentials, and also develop a time-dependent wave packet approximation which we illustrate for the Morse potential. These time-dependent solutions allow us to determine the stability properties of the previously studied stationary solutions. On the basis of these developments we then reconsider the Bohm approach to the NM treatment of the position measurement process.