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Nonequilibrium steady state in open quantum systems: Influence action, stochastic equation and power balance

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HIGHLIGHTS

- Nonequilibrium steady state (NESS) for interacting quantum many-body systems.
- Derivation of stochastic equations for quantum oscillator chain with two heat baths.
- Explicit calculation of the energy flow from one bath to the chain to the other bath.
- Power balance relation shows the existence of NESS insensitive to initial conditions.
- Functional method as a viable platform for issues in quantum thermodynamics.

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ABSTRACT

The existence and uniqueness of a steady state for nonequilibrium systems (NESS) is a fundamental subject and a main theme of research in statistical mechanics for decades. For Gaussian systems, such as a chain of classical harmonic oscillators connected at each end to a heat bath, and for classical anharmonic oscillators under specified conditions, definitive answers exist in the form of proven theorems. Answering this question for quantum many-body systems poses a challenge for the present. In this work we address this issue by deriving the stochastic equations for the reduced system with self-consistent backaction from the two baths, calculating the energy flow from one bath to the chain to the other bath, and exhibiting a power balance relation in the total (chain + baths) system which testifies to the existence of a NESS in

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this system at late times. Its insensitivity to the initial conditions of the chain corroborates to its uniqueness. The functional method we adopt here entails the use of the influence functional, the coarse-grained and stochastic effective actions, from which one can derive the stochastic equations and calculate the average values of physical variables in open quantum systems. This involves both taking the expectation values of quantum operators of the system and the distributional averages of stochastic variables stemming from the coarse-grained environment. This method though formal in appearance is compact and complete. It can also easily accommodate perturbative techniques and diagrammatic methods from field theory. Taken all together it provides a solid platform for carrying out systematic investigations into the nonequilibrium dynamics of open quantum systems and quantum thermodynamics.

1. Introduction

Nonequilibrium stationary states (NESS) play a uniquely important role in many-body systems in contact with two or more heat baths at different temperatures, similar in importance to the *equilibrium state* of a system in contact with one heat bath which is the arena for the conceptualization and utilization of the canonical ensemble in statistical thermodynamics. The statistical mechanics [1] and thermodynamics [2] of open systems[1] in NESS have been the focus of investigation into the important features of nonequilibrium processes of theoretical interests, such as providing the context for the celebrated classical and quantum fluctuation theorems, and acting as the fountainhead of a new field known as quantum thermodynamics [7,8], where the laws of classical thermodynamics are now scrutinized for smaller quantum systems (e.g., [9,10]), and with a wide range of practical applications, extending from physics and chemistry to biology.

For classical many body systems the existence and uniqueness of NESS is a fundamental subject and a main theme of research by mathematical physicists in statistical mechanics for decades. For Gaussian systems (such as a chain of harmonic oscillators with two heat baths at the two ends of the chain) [11] and anharmonic oscillators under general conditions [12] there are definitive answers in the form of proven theorems. Answering this question for quantum many body systems is not so straightforward and poses a major challenge for the present. For quantum many body systems a new direction of research is asking whether *closed* quantum systems can come to equilibrium and

[1] Defined in a broader sense (A) an open system is one where some of its information is difficult or impossible to obtain or retrieve, or is coarse-grained away by design or by necessity, both in theoretical and practical terms, the latter referring to the limited capability of the measuring agent or the precision level of instrumentation. The more specific sense (B) used in nonequilibrium statistical mechanics [3] emphasizing the influence of a system's environment on its dynamics goes as follows: Start with a closed system comprising of two subsystems S_1 and S_2 with some interaction between the two, one can express the dynamics of S_1 including that of S_2 in terms of an integral differential equation. If one subsystem S_2 contains an overwhelmingly large number of degrees of freedom than the other, we call S_2 an environment E of S_1. The influence of E on S_1 is called the backaction. When the environment can be characterized by thermodynamic parameters it is called a heat bath at temperature T or a matter reservoir with chemical potential μ, etc. When a great deal of microscopic information of S_2 is discarded or coarse-grained, as is the case when it is described only by a few macroscopic parameters, the effect of the environment can be characterized by noise and fluctuations [4], and their backaction on the system show up in the "reduced" system's dynamics as dissipation [5], diffusion (quantum diffusion is responsible for the decoherence [6] of quantum phase information). An open system thus carries the influence or the backaction of its environment. Oftentimes the main task in the treatment of open systems is to find the influence of the environment on the subsystem. Defined in this sense (B) it is synonymous with "reduced" system—with the burden of explanation now shifted to what "reduced" entails operationally.