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Construction of the conserved ζ via the effective action for perfect fluids

Junpu Wang

Department of Physics and ISCAP, Columbia University, New York, NY 10027, United States

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ABSTRACT

We consider the problem of how to construct the curvature perturbation ζ, which is expected to be time independent on super-horizon scales to nonlinear level; in particular we concentrate on the case where the existence of such conserved ζ is guaranteed: a universe dominated by a perfect fluid with adiabatic pressure. We have used a low energy/long wavelength effective action to model the fluid sector and coupled it to the Einstein gravity. This setup enables us to verify explicitly the assumption of "local homogeneity and isotropy" used in previous literature. As a corollary, we also show that the nonlinearly defined graviton field γ_{ij} is conserved outside the horizon in the same manner as ζ is.

1. Introduction

Despite the significant success of linear perturbation theory in cosmology over the last few decades, more and more efforts have been devoted in recent years to understanding the cosmological perturbations beyond the linear level—both their origins and evolutions. This is partially motivated by the extraordinary improvement in the accuracy of experiments. For instance, although the result from PLANCK implied the absence of primordial non-Gaussianities, the study on the afterward standard cosmology, such as on the large scale structures, calls for better understanding of the non-linear cosmological perturbations on both the conceptual and the technical/computational level. On the other hand, the observables in cosmology are usually related to *gauge invariant* quantities, which are rather ambiguous at linear order (since a gauge invariant quantity multiplied by any function of

E-mail address: junpu@phys.columbia.edu.

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$a(t)$, $H(t)$, etc. is still gauge invariant at linear order). So the study of nonlinear perturbations might give us hints on how observables are related to fundamental physical degrees of freedom.

Among these quantities, enormous attention has been paid to the (linear) curvature perturbation ζ on uniform Hubble slicings [1]. Equivalently it can thought as the linear scalar metric perturbation on uniform-density hyper-surfaces in [2] and was shown that it is time-independent (conserved) outside the horizon (cf. for instance [3]). Many studies have been done recently seeking to extend the construction of a conserved ζ to non-linear level in different contexts. Roughly speaking, there are three classes of methods employed to achieve this, which are summarized as follows:

- The first class is the standard perturbative approach—a perturbative expansion in fields [4]. It is straightforward and the equations governing the evolution of ζ are valid on all scales. However, the price to pay is that the equations usually become cumbersome once one goes beyond the linear order in the fields and there is a lack of systematic ways to demonstrate the conservation of ζ on super-horizon scales to arbitrary orders in the field expansion.
- The second class of approaches instead focuses on the spatial gradient expansion—that is, an expansion in powers of $\sigma \equiv k/aH$, where k denotes the wavenumber of each Fourier mode. At leading order of this expansion, it corresponds to the separate universe approach. Refs. [5,6] investigated the cases with an inflating background. There the authors showed that in single field inflation models, ζ can be defined as the (only) dynamical scalar metric perturbation in the unitary gauge (in which the matter field is demanded to be unperturbed), and that such a ζ was conserved to *all* orders in fields on super-horizon scales. The proofs were based on a Lagrangian formalism, for the matter content in these cases was specified by explicit matter actions. On the other hand, the cases with a general FRW background were studied using the formalism developed in [7–9] (known as the δN formalism). It was shown that the conservation of the nonlinear ζ on large scales was assured if the pressure of the matter content was only a function of the energy density or if the matter content could be modeled by a single scalar field. As opposed to the aforementioned field theoretic formalism used in inflation scenarios, these analyses relied on the field equations of motion; in the context of the Einstein gravity the energy conservation alone would be sufficient.
- A third approach was proposed in [10,11] by invoking the purely geometrical description of the curvature perturbation. The covariant quantity constructed there satisfied an exact, non-perturbative, valid-to-all-scale conservation equation and would eventually reduce to the usual ζ on large scales. The equivalence between this covariant formalism and the δN formalism was established in [12,13].

In this note, we will present a systematic way of constructing the conserved curvature perturbation ζ in a universe dominated by a perfect fluid to all orders in the fields. A low energy effective action for an ordinary perfect fluid [14,15] will be employed to model the matter content. So our method here is parallel to that used in the single field inflation cases [5,6]. On the other hand, although this scenario was investigated to some extent in previous literature, we believe that our approach has its own merits in deepening the understanding of this issue from some other aspects: It was crucial for Refs. [7,8,16] to assume that the universe, when it is probed by a sufficiently long wavelength mode, can approximately be thought of as being composed of homogeneous and isotropic patches, each of which looks like an FRW universe. With this very strong assumption, the analysis there can be applied to rather general situations, without even specifying the gravity theory [7,8,16]. While in our method, since we specify that the fluid is coupled to the Einstein gravity, we can, in principle, test the assumptions of "local homogeneity and isotropy" using the knowledge of the Lagrangian of this dynamical system. Indeed, by applying the Einstein field equations, we can actually show that there exists such a coordinate (or, a gauge choice) that "local homogeneity and isotropy" is manifest. Furthermore our approach enables us to construct the vector and tensor counterparts of ζ—i.e. the nonlinear vector and tensor perturbations that are time-independent outside the horizon.[1]

[1] The conservation of the vector field does not contradict the usual intuition that vector modes decay in the absence of anisotropic stress tensor. The time-independent piece of the vector perturbation is of leading order in the spatial gradient expansion. As we will see later, due to the symmetry requirements of a fluid, any constant (in time) transverse vector field