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Magnon–magnon interactions in $O(3)$ ferromagnets and equations of motion for spin operators

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ABSTRACT

The method of equations of motion for spin operators in the case of $O(3)$ Heisenberg ferromagnet is systematically analyzed starting from the effective Lagrangian. It is shown that the random phase approximation and the Callen approximation can be understood in terms of perturbation theory for type B magnons. Also, the second order approximation of Kondo and Yamaji for one dimensional ferromagnet is reduced to the perturbation theory for type A magnons. An emphasis is put on the physical picture, i.e. on magnon–magnon interactions and symmetries of the Heisenberg model. Calculations demonstrate that all three approximations differ in manner in which the magnon–magnon interactions arising from the Wess–Zumino term are treated, from where specific features and limitations of each of them can be deduced.

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1. Introduction

The study of magnon–magnon interactions is a problem with long history [1] whose complexity initiated development of numerous analytical tools. Its importance goes far beyond pure academic interest since with the advent of new materials, especially those connected to the high-temperature superconductors [2,3], the question of magnon–magnon interactions and their influence on thermodynamic properties of ordered magnets became an urgent one to solve. This amounts not only

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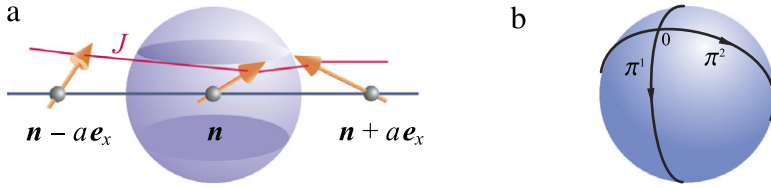


Fig. 1. (color online) Different perspectives on dynamical degrees of freedom and magnon–magnon interactions in $O(3)$ ferromagnets:

(a) In spin-operator-based approaches, interactions are viewed as a consequence of nontrivial algebra and Hilbert space of spin operators coupled through exchange integral J . [One-dimensional lattice of localized $S = 1/2$ spins is displayed for clarity.] (b) In EFT, magnons are considered as a set of (local) coordinates on the sphere $S^2 = O(3)/O(2)$ and magnon–magnon interactions are dictated by the underlying geometry of S^2 , the non-Abelian character of $O(3)$ and the existence of conserved charges in ferromagnetic ground state.

to finding efficient calculation techniques, but also understanding how or why certain approximations work and what are their domains of applicability. With this in mind, it is useful to study simpler models [4]. In this case, they reveal some features generic to the wide class of spin systems otherwise concealed in more complicated ones containing second and third neighbor interaction, various types of frustration and anisotropies, random impurities etc. Much like 2D Ising model, due to several well established and exact results, the (quantum) $O(3)$ Heisenberg ferromagnet has come to be a prototypical model for testing diverse theoretical and numerical methods.

Nearly all theories addressing the problem of magnon–magnon interactions are spin-operator oriented in the sense that all calculations are based on the operators used to define the Heisenberg model (see Fig. 1). The most straightforward way to incorporate the interaction effects is to express the localized spins in terms of bosonic/fermionic operators directly in the Heisenberg Hamiltonian or to use the coherent-state path integral (see e.g. [5,6]). The original spin Hamiltonian is afterwards interpreted as describing the system of interacting bosons/fermions, to which standard perturbation theory, or suitable mean-field approximations (MFA), may be applied (see [1,5–8] and references therein). Although techniques build upon boson/fermion representations of spin operators provide an important insight into the behavior of magnetic systems, they suffer from a universal disadvantage: The $su(2)$ algebra of spin operators and the dynamics of the spin system are fully satisfied only with exact boson/fermion Hamiltonian and corresponding Hilbert space. Accordingly, any approximation in the boson/fermion Hamiltonian, such as approximate expression for localized spins or some mean-field approximation, destroys the spin nature of ($S^\pm$, S^z) operators, often in an uncontrolled manner.

A different way to deal with magnon–magnon interactions is based on the equations of motion (EOM) for spin operators [9]. Aside from the fact that the EOM for spin operators are practically unsolvable for general Heisenberg-type Hamiltonian, the analysis through EOM gets more involved giving the fact that spin operators present basic dynamical variables and charge densities at the same time [10]. Despite all that, the method of equations of motion, especially in temperature-Green's function (TGF) disguise (see [11] for a recent review and original references), gained quite a popularity. There are several reasons for this. First, its predictions for thermodynamic properties of Heisenberg-type magnets agree with Monte Carlo simulations [11] and with experimental results (see e.g. [12–16] and references therein). Equally important is the flexibility of the method making it easily adjustable to systems with complex [17–21] or low-dimensional lattices [22–29], second and third-neighbor interaction or anisotropies [12,14,30–34], which is of great importance when studying real compounds. Also, the TGF method is recognized as useful in theories of diluted magnetic systems [35,36], nuclear spin order in quantum wires [37], multiferroics models [38,39] and even in theories of itinerant electron systems where Heisenberg Hamiltonian appears as an intermediate effective model [21,40,41]. The approximations made directly in the equations of motion, known as the decoupling schemes (DS) in the TGF jargon [11], enable one to solve linearized system of EOM-s thereby determining the magnon spectrum, correlation functions etc. By suitably choosing the parameters of linearization [42,43], more or less satisfactory results may be obtained. Even though TGF method is extensively exploited in contemporary research (see the papers quoted earlier in this

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