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Quantum-corrected finite entropy of noncommutative acoustic black holes

M.A. Anacleto [a], F.A. Brito [a,b,*], G.C. Luna [a], E. Passos [a], J. Spinelly [c]

[a] *Departamento de Física, Universidade Federal de Campina Grande, Caixa Postal 10071, 58109-970 Campina Grande, Paraíba, Brazil*

[b] *Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brazil*

[c] *Departamento de Física-CCT, Universidade Estadual da Paraíba, Juvêncio Arruda S/N, Campina Grande, PB, Brazil*

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ABSTRACT

In this paper we consider the generalized uncertainty principle in the tunneling formalism via Hamilton–Jacobi method to determine the quantum-corrected Hawking temperature and entropy for $2 + 1$-dimensional noncommutative acoustic black holes. In our results we obtain an area entropy, a correction logarithmic in leading order, a correction term in subleading order proportional to the radiation temperature associated with the noncommutative acoustic black holes and an extra term that depends on a conserved charge. Thus, as in the gravitational case, there is no need to introduce the ultraviolet cut-off and divergences are eliminated.

1. Introduction

The study of acoustic black holes was proposed in 1981 by Unruh [1] and has been extensively studied in the literature [2–4]. Acoustic black holes was found to possess many of the fundamental properties of black holes in general relativity and has been developed to investigate the Hawking radiation and other phenomena for understanding quantum gravity. Thus, many fluid systems have been investigated on a variety of analog models of acoustic black holes, including gravity wave [5],

* Corresponding author at: Departamento de Física, Universidade Federal de Campina Grande, Caixa Postal 10071, 58109-970 Campina Grande, Paraíba, Brazil.

E-mail addresses: anacleto@df.ufcg.edu.br (M.A. Anacleto), fabrito@df.ufcg.edu.br (F.A. Brito), gabrielacluna@hotmail.com (G.C. Luna), passos@df.ufcg.edu.br (E. Passos), jspinelly@uepb.edu.br (J. Spinelly).

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water [6], slow light [7], optical fiber [8] and electromagnetic waveguide [9]. The models of superfluid helium II [10], atomic Bose–Einstein condensates [11,12] and one-dimensional Fermi degenerate noninteracting gas [13] have been proposed to create an acoustic black hole geometry in the laboratory. In this case the theory of the electrons in a constant magnetic field, projected to the lowest Landau level, is naturally thought of as a noncommutative field theory [14]. A relativistic version of acoustic black holes has been presented in [15,16].

In Ref. [17] was investigated $(1+1)$-dimensional acoustic black hole entropy by the brick-wall method. In order to obtain a finite result, they had to introduce the ultraviolet cut-off. So their calculation suggested that analog black hole entropy has the cut-off problem similar to that of gravitational black hole entropy. More recently in [18] the author uses transverse modes in order to cure the divergences.

The study on the statistical origin of black hole entropy has been extensively explored by several authors — see for instance [19–21]. In Ref. [22], Kaul and Majumdar compute the lowest order corrections to the Bekenstein–Hawking entropy. They find that the leading correction is logarithmic, with

$$S \sim \frac{A}{4G} - \frac{3}{2}\ln\left(\frac{A}{4G}\right) + const. + \cdots$$

on the other hand, Carlip in Ref. [23] compute the leading logarithmic corrections to the Bekenstein–Hawking entropy and shows that the logarithmic correction is identical to that of Kaul and Majumdar, plus corrections that depend on conserved charges, as

$$S \sim \frac{A}{4G} - \frac{3}{2}\ln\left(\frac{A}{4G}\right) + \ln[F(Q)] + const. + \cdots$$

where $F(Q)$ is some function of angular momentum and other conserved charges.

The brick-wall method proposed by G. 't Hooft has been used for calculations on the black hole, promoting the understanding of the origin of black hole entropy. According to G. 't Hooft, black hole entropy is just the entropy of quantum fields outside the black hole horizon. However, when one calculates the black hole statistical entropy by this method, to avoid the divergence of states density near black hole horizon, an ultraviolet cut-off must be introduced. The other related idea in order to cure the divergences is to consider models in which the Heisenberg uncertainty relation is modified, for example in one dimensional space, as

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \alpha^2(\Delta p)^2\right),$$

which shows that there exists a minimal length $\Delta x \geq \hbar\alpha$, where Δx and Δp are uncertainties for position and momentum, respectively, and α is a positive constant which is independent of Δx and Δp. A commutation relation for the generalized uncertainty principle (GUP) can be written as $[x, p]_{GUP} = i\hbar(1 + \alpha^2 p^2)$, where x and p are the position and the momentum operators, respectively. Thus, using the modified Heisenberg uncertainty relation the divergence in the brick-wall model are eliminated as discussed in [24]. The statistical entropy of various black holes has also been calculated via corrected state density of the GUP [25]. Thus, the results show that near the horizon quantum state density and its statistical entropy are finite. In [26] a relation for the corrected states density by GUP has been proposed

$$dn = \frac{d^3x d^3p}{(2\pi)^3} e^{-\lambda p^2},$$

where $p^2 = p^i p_i$, and λ plays the role of the Planck scale and in a fluid at high energy regimes.

Recently, the authors in [27] using a new equation of state density due to GUP [28], the statistical entropy of a $2+1$-dimensional rotating acoustic black hole has been analyzed. It was shown that using the quantum statistical method the entropy of the rotating acoustic black hole was calculated, and the Bekenstein–Hawking area entropy of acoustic black hole and its correction term was obtained. Therefore, considering the effect due to GUP on the equation of state density, no cut-off is needed [29] and the divergence in the brick-wall model disappears.