Annals of Physics 362 (2015) 449–472

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Diagonal ordering operation technique applied to Morse oscillator

Dušan Popov [a,*], Shi-Hai Dong [b], Miodrag Popov [c]

[a] Politehnica University Timisoara, Department of Physical Foundations of Engineering, Bd. V. Parvan No. 2, 300223 Timisoara, Romania

[b] CIDETEC, Instituto Politecnico Nacional, Unidad Profesional Adolfo Lopez Mateos, Mexico D.F. 07700, Mexico

[c] Politehnica University Timisoara, Department of Steel Structures and Building Mechanics, Traian Lalescu Street, No. 2/A, 300223 Timisoara, Romania

ARTICLE INFO

Article history:
Received 19 May 2015
Accepted 11 August 2015
Available online 18 August 2015

Keywords:
Normally ordered product
Morse oscillator
Coherent state
Mixed spectrum

ABSTRACT

We generalize the technique called as the integration within a normally ordered product (IWOP) of operators referring to the creation and annihilation operators of the harmonic oscillator coherent states to a new operatorial approach, i.e. the diagonal ordering operation technique (DOOT) about the calculations connected with the normally ordered product of generalized creation and annihilation operators that generate the generalized hypergeometric coherent states. We apply this technique to the coherent states of the Morse oscillator including the mixed (thermal) state case and get the well-known results achieved by other methods in the corresponding coherent state representation. Also, in the last section we construct the coherent states for the continuous dynamics of the Morse oscillator by using two new methods: the discrete–continuous limit, respectively by solving a finite difference equation. Finally, we construct the coherent states corresponding to the whole Morse spectrum (discrete plus continuous) and demonstrate their properties according the Klauder's prescriptions.

* Corresponding author.
E-mail addresses: dusan_popov@yahoo.co.uk, dusan.popov@upt.ro (D. Popov).

http://dx.doi.org/10.1016/j.aop.2015.08.011

1. Introduction

After introducing the concept of canonical coherent states (CSs), associated with the one dimensional harmonic oscillator (HO-1D) by Schrödinger in 1926 [1], followed decades in which this concept has aroused great scientific interest. Since the second half of the last century it has begun to become clear advantages and applications of this formalism, so they have built CSs associated with different kind of Hamiltonians. Also, scientific literature was enriched with different kinds of CSs for the anharmonic oscillators, either as the lowering operator eigenstate (Barut–Girardello), i.e. BG-CSs or by applying the displacement operator on a ground state (Klauder–Perelomov) or CSs of the Gazeau–Klauder kind, including the nonlinear CSs, squeezed states and deformed CSs. Its applicability has been considerably expanded to different fields beginning with condensed matter physics and mathematical physics to signal theory and quantum information. This can be seen in numerous books or review papers on the CSs and their applications [2–6] (and the references therein). Moreover, in various CSs, an interesting category of CSs is the generalized hypergeometric coherent states (GH-CSs), firstly introduced by Appl and Schiller [7]. These states were applied to the mixed (thermal) states of the pseudoharmonic oscillator in our previous paper [8]. On the other hand, in different calculations involving CSs it is necessary to achieve commutation of operators or their ordering according to certain rules. It is known that the integration within an ordered product (IWOP) technique for Bose operators, refers to the CSs of the HO-1D (see, [9] and the references therein), or so called double dot operation : : (see [10]), which consists of applying the normal ordered IWOP rules without taking into account the commutation relation between the annihilation and creation operators. Recently, we have introduced a new approach of ordering the operators called the *diagonal ordering operation technique* (DOOT) [11] and noted it by the symbol # #. This technique can be considered as a generalization of the Fan's IWOP technique. It is showed that the DOOT technique can be formulated and used not only for the CSs of the HO-1D, but also can be generalized to other types of nonlinear CSs, which are the particular cases of more general generalized hypergeometric Barut–Girardello coherent states (HG-BG-CSs). The aim of this paper is to apply the DOOT to the Morse oscillator CSs having in mind that this oscillator has a finite number of bound levels and implicitly to rediscover the results obtained by traditional methods in our previous work [12].

The paper is organized as follows. In Section 2 we present some general properties of the Morse oscillator. In Section 3 we briefly review some basic elements and main results of the DOOT for the HG-BG-CSs. In Section 4 we apply the DOOT to the BG-CSs of the discrete part of Morse oscillator spectrum and rediscover the corresponding results obtained by other methods [12]. In Section 5 we present a practical approximation to some obtained formulas about the mixed (thermal) states of Morse oscillator. In Section 6 we propose and construct the CSs for the continuous spectrum of the Morse oscillator using an original method. In the last Section we present some arguments to illustrate the usefulness of the DOOT, i.e. the DOOT can be applied, with some particularities, not only to the quantum systems with infinite number of bound state levels but also to the case of finite number of bound state levels, e.g. the Morse oscillator being a typical example.

2. General properties of the Morse oscillator

Let us briefly recall the main elements of the Morse oscillator. With the same notations as in our previous paper [12], the radial part of the Morse Hamiltonian is given by

$$H = -\frac{\hbar^2}{2m_{red}}\frac{d^2}{dr^2} + V_M(r) \equiv -\frac{\hbar^2}{2m_{red}}\frac{d^2}{dr^2} + D_e\left(1 - \exp\left[-\alpha\left(r - r_e\right)\right]\right)^2, \tag{2.1}$$

where m_{red} is the reduced mass, D_e the dissociation energy, r_e the inter nuclear distance, and α the exponential parameter connected by the fundamental vibrational frequency ω through the relation $\alpha = \omega\sqrt{m_{red}/2D_e}$.

The Schrödinger equation for the Morse oscillator can be solved exactly, the vibrational energy eigenvalues are

$$E_n = \varepsilon\left(\frac{1}{2} + n - \frac{1}{N}n^2\right), \tag{2.2}$$