Annals of Physics 362 (2015) 511–520

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Monogamy of αth power entanglement measurement in qubit systems

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ARTICLE INFO

Article history:
Received 15 July 2015
Accepted 21 August 2015
Available online 28 August 2015

Keywords:
Monogamy
Entanglement measurement

ABSTRACT

In this paper, we study the αth power monogamy properties related to the entanglement measure in bipartite states. The monogamy relations related to the αth power of negativity and the Convex-Roof Extended Negativity are obtained for N-qubit states. We also give a tighter bound of hierarchical monogamy inequality for the entanglement of formation. We find that the GHZ state and W state can be used to distinguish both the αth power of the concurrence for $0 < \alpha < 2$ and the αth power of the entanglement of formation for $0 < \alpha \leq \frac{1}{2}$. Furthermore, we compare concurrence with negativity in terms of monogamy property and investigate the difference between them.

1. Introduction

Multipartite entanglement is an important physical resource in quantum mechanics, which can be used in quantum computation, quantum communication and quantum cryptography. One of the most surprising phenomenon for multipartite entanglement is the monogamy property, which may be as fundamental as the no-cloning theorem [1–4]. The monogamy property can be interpreted as the amount of entanglement between A and B, plus the amount of entanglement between A and C, cannot be greater than the amount of entanglement between A and the pair BC. Monogamy property have been considered in many areas of physics: one can estimate the quantity of information captured by an eavesdropper about the secret key to be extracted in quantum cryptography [3,5], the frustration effects observed in condensed matter physics [6], even in black-hole physics [7,8].

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http://dx.doi.org/10.1016/j.aop.2015.08.022
0003-4916/

Historically, monogamy property of various entanglement measure have been discovered. Coffman et al. first considered three qubits A, B and C which may be entangled with each other [2], who showed that the squared concurrence $\mathcal{C}^2$ follows this monogamy inequality. Osborne et al. proved the squared concurrence follows a general monogamy inequality for N-qubit system [3]. Analogous to the Coffman–Kundu–Wootters (CKW) inequality, Ou et al. proposed the monogamy inequality holds in terms of squared negativity $\mathcal{N}^2$ [9]. Kim et al. showed that the squared convex-roof extended negativity $\widetilde{\mathcal{N}}^2$ follows monogamy inequality [10]. Oliveira et al. and Bai et al. investigated entanglement of formation (EoF) and showed that the squared EoF E^2 follows the monogamy inequality [11,12]. A natural question is why those monogamy property above are squared entanglement measure? In fact, Zhu et al. showed that the αth power of concurrence $\mathcal{C}^\alpha$ ($\alpha \geq 2$) and the αth power of entanglement of formation E^α ($\alpha \geq \sqrt{2}$) follow the general monogamy inequalities [13]. Sometimes, we can view the coefficient α as a kind a of assigned weight to regulate the monogamy property [14,15].

In this paper, we study the monogamy relations related to αth power of some entanglement measures. We show that the αth power of negativity $\mathcal{N}^\alpha$ and the αth power of convex-roof extended negativity (CREN) $\widetilde{\mathcal{N}}^\alpha$ follows the hierarchical monogamy inequality for $\alpha \geq 2$ [16]. From the hierarchical monogamy inequality, the general monogamy inequalities related to $\mathcal{N}^\alpha$ and $\widetilde{\mathcal{N}}^\alpha$ are obtained for N-qubit states. We find that the GHZ state and W state can be used to distinguish the $\mathcal{C}^\alpha$ for $0 < \alpha < 2$, which situation was not clear in Zhu et al.'s paper [13]. We also find that the GHZ state and W state can be used to distinguish the αth power of EoF for $0 < \alpha \leq \frac{1}{2}$. The hierarchical monogamy inequality for E^α is also discussed, which improved Bai et al.'s result [16,12].

This paper is organized as follows. In Section 2,we study the monogamy property of αth power of negativity. In Section 3, we discuss the monogamy property of αth power of CREN. In Section 4, we study the monogamy property of αth power of EoF. In Section 5, we compare the monogamy property of different entanglement measures. We summarize our results in Section 6.

2. Monogamy of αth power of negativity

Given a bipartite state ρ_{AB} in the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Negativity is defined as [17]:

$$\mathcal{N}(\rho_{AB}) = \frac{\|\rho_{AB}^{T_A}\| - 1}{2}, \tag{1}$$

where $\rho_{AB}^{T_A}$ is the partial transpose with respect to the subsystem A, $\|X\|$denotes the trace norm of X, i.e $\|X\| \equiv Tr\sqrt{XX^\dagger}$. Negativity is a *computable* measure of entanglement, and which is a convex function of $\rho_{AB} \cdot \mathcal{N}(\rho_{AB}) = 0$ if and only if ρ_{AB} is separable for the $2 \otimes 2$ and $2 \otimes 3$ systems [18]. For the purposes of discussion, we use following definition of negativity:

$$\mathcal{N}(\rho_{AB}) = \|\rho_{AB}^{T_A}\| - 1. \tag{2}$$

For any maximally entangled state in two-qubit system, this definition of negativity is equal to 1.

For a bipartite pure state $|\psi_{AB}\rangle$, the concurrence is defined as:

$$\mathcal{C}(|\psi_{AB}\rangle) = \sqrt{2[1 - Tr(\rho_A^2)]} = 2\sqrt{\det \rho_A}, \tag{3}$$

where ρ_A is the reduced density matrix of subsystem A. For a mixed state ρ_{AB}, the concurrence can be defined as:

$$\mathcal{C}(\rho_{AB}) = \min \sum_i p_i \mathcal{C}(|\psi_{AB}^i\rangle), \tag{4}$$

where the minimum is taken over all possible pure state decompositions $\{p_i, \psi_{AB}^i\}$ of ρ_{AB}.

The next *lemma* builds a relationship between negativity and concurrence in a $2 \otimes m \otimes n$ system ($m \geq 2, n \geq 2$).

Lemma 1. *For a pure state $|\psi\rangle_{ABC}$ in a $2 \otimes m \otimes n$ system ($m \geq 2, n \geq 2$), the negativity of bipartition $A|BC$ is equal to its concurrence: $\mathcal{N}_{A|BC} = \mathcal{C}_{A|BC}$, where $\mathcal{N}_{A|BC} = \mathcal{N}(|\psi_{ABC}\rangle)$ and $\mathcal{C}_{A|BC} = \mathcal{C}(|\psi_{ABC}\rangle)$.*