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# Magnetically charged regular black hole in a model of nonlinear electrodynamics

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ABSTRACT

We obtain a magnetically charged regular black hole in general relativity. The source to the Einstein field equations is nonlinear electrodynamic field in a physically reasonable model of nonlinear electrodynamics (NED). "Physically" here means the NED model is constructed on the basis of three conditions: the Maxwell asymptotic in the weak electromagnetic field limit; the presence of vacuum birefringence phenomenon; and satisfying the weak energy condition (WEC). In addition, we analyze the thermodynamic properties of the regular black hole in two ways. According to the usual black hole thermodynamics, we calculate the heat capacity at constant charge, from which we know the smaller black hole is more stable. We also employ the horizon thermodynamics to discuss the thermodynamic quantities, especially the heat capacity at constant pressure.



## 1. Introduction

The first well-known model of nonlinear electrodynamics (NED) is the Born–Infeld theory (BI), which is proposed to obtain a finite electron self-energy [1]. Heisenberg and Euler found that due to the presence of virtual charged particles the one-loop quantum correction in quantum electrodynamics will give nonlinear contribution [2]. Not only that, the virtual particles will result in "polarization of the vacuum". In this case, the vacuum behaves like a polarizable continuum and should exhibit the phenomenon of birefringence [3–5]. These effects can be observed in experiments such as PVLAS [6]

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and BMV [7] and the experimental results can put some restrictions on the parameters introduced in the NED models. Vacuum birefringence is a nonlinear effect. Therefore, we expect that in a physically reasonable model of NED, there should exist the effect of vacuum birefringence. In this sense, the usual BI theory is not a physically allowable NED model because of the absence of vacuum birefringence. However, in the generalized BI model with two parameters the vacuum birefringence may exist [8]. The nonlinear effects in electrodynamics are significant only for strong electromagnetic fields. In weak field case, NED should return back to the conventional Maxwell theory, just like the weak field approximation of general relativity gives the Newtonian mechanics. Thus, we expect that this is another requirement for a physically reliable NED model.

Recently, due to the natural emergence in string theories [9–11], NED is coming into view in gravitational theories as source. It is well-known that by minimal coupling of Maxwell electromagnetic fields to gravity, the Reissner–Nordström (RN) black hole can be obtained. Similarly, more interesting black hole solutions can be derived through minimal coupling to gravity of nonlinear electromagnetic fields, such as BI and BI-like electromagnetic fields [12,13], logarithmic electromagnetic field [14], power electromagnetic field [15] and exponential electromagnetic field [16].

In this letter, we shall investigate regular black hole solution derived in general relativity coupled to NED. The first example of a regular black hole was constructed by Bardeen in 1968 [17]. Nearly thirty years later, Ayón-Beato et al. reobtained the Bardeen black hole by describing it as the gravitational field of a kind of nonlinear magnetic monopole [18]. Similarly, many other regular black holes can also be constructed by introducing nonlinear electromagnetic sources [19–23]. There are also other types of regular/nonsingular black holes with different origins. For more detailed description of regular black holes, one can refer to the paper by Lemos et al. [24] and references therein. We should stress that there is a theorem which asserts that the existence of electrically charged, static, spherically symmetric solutions with a regular center is forbidden, while the existence of the solutions with magnetic charges is feasible, if the NED model contains the Maxwell theory as its weak approximations [22]. Thus, we only concern with the magnetically charged regular black hole with the special emphasis put on its thermodynamics and stability.

The paper is arranged as follows: in Section 2 we simply introduce the NED model and its origin. In Section 3 we will solve the Einstein field equations to obtain the regular black hole solution and analyze the geometric structure. In Section 4 we verify that the nonlinear electromagnetic field in this NED model satisfies the WEC. Then we calculate the thermodynamic quantities, such as temperature, the heat capacity at constant charge and the heat capacity at constant pressure from which we can discuss the local stability of the regular black holes in Section 5. We will make some concluding remarks in Section 6.

## 2. The NED model

Firstly we simply introduce the model of NED proposed by Kruglov [25], which can produce the vacuum birefringence phenomenon. The Lagrangian is given by

$$\mathcal{L} = \mathcal{F} + \frac{a\mathcal{F}}{2\beta\mathcal{F}+1} - \frac{\gamma}{2}\mathcal{G}^2, \tag{1}$$

where $a$ is a dimensionless parameter and $\beta,\ \gamma$ are parameters with the dimensions of $[L^2]$. $\mathcal{F} = F_{\mu\nu}F^{\mu\nu},\ \mathcal{G} = F_{\mu\nu}\tilde{F}^{\mu\nu}$ are two Lorentz invariants with $\tilde{F}_{\mu\nu} = 1/2\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength.

It is shown that in a constant and uniform external magnetic field $\mathbf{B}_0$, the indexes of refraction with different polarizations of electromagnetic waves are

$$n_\parallel = \sqrt{1 + \frac{\gamma B_0^2(\beta B_0^2+1)^2}{a+(\beta B_0^2+1)^2}}, \qquad n_\perp = 1. \tag{2}$$

This means that the electromagnetic waves with different polarizations have different velocities and thus the vacuum birefringence is present. However, one can easily see that this NED model cannot