Annals of Physics 362 (2015) 538–550

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Geometric curvature and phase of the Rabi model

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ARTICLE INFO

Article history:
Received 5 June 2015
Accepted 18 August 2015
Available online 1 September 2015

Keywords:
Geometric curvature
Geometric phase
Rabi model

ABSTRACT

We study the geometric curvature and phase of the Rabi model. Under the rotating-wave approximation (RWA), we apply the gauge independent Berry curvature over a surface integral to calculate the Berry phase of the eigenstates for both single and two-qubit systems, which is found to be identical with the system of spin-1/2 particle in a magnetic field. We extend the idea to define a vacuum-induced geometric curvature when the system starts from an initial state with pure vacuum bosonic field. The induced geometric phase is related to the average photon number in a period which is possible to measure in the qubit–cavity system. We also calculate the geometric phase beyond the RWA and find an anomalous sudden change, which implies the breakdown of the adiabatic theorem and the Berry phases in an adiabatic cyclic evolution are ill-defined near the anti-crossing point in the spectrum.

1. Introduction

The Pancharatnam–Berry phase [1,2], or most commonly known as Berry phase, has provided us a very deep insight on the geometric structure of quantum mechanics. It is worthwhile to note that Berry phase has attracted considerable interests in quantum theory on account of giving rise to interesting observable physical phenomena, implementing the operation of a universal quantum logic gate in quantum computing [3–6]. The most significant characteristic for the concept of Berry phase is the existence of a continuous parameter space in which the state of the system varies slowly

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http://dx.doi.org/10.1016/j.aop.2015.08.026
0003-4916/

along a closed cycle [2,7,8]. In particular, various extensions of the phase have been considered [9], such as geometric phases for mixed states [10], for open systems [11], and with a quantized driving field [12–17]. Furthermore, the concept of a geometric phase has been generalized to the case of noneigenstates, which is applicable to both linear and nonlinear quantum systems [18]. In the linear case, the geometric phase reduces to a statistical average of Berry phases for the eigenstates, weighted by the probabilities that the system finds itself in the eigenstates.

The Jaynes–Cummings (JC) model [19], or the quantum Rabi model [20] with the rotating wave approximation (RWA) that describes a spin-1/2 particle interacting with a single mode quantized field, plays an important role in the cavity quantum electrodynamics. So Fuentes-Guridi et al. [12] calculated the Berry phase of the JC model considering the quantum nature of the field. A recent work [21] made a comparison study between the Berry phases of the JC model with and without the RWA where the Berry phase for the ground state varied from zero under the RWA to nonzero values beyond the RWA. In addition, by means of two unitary transformations, the Berry phase of the JC model without the RWA, is presented in a simple and straightforward analytical method [22]. Clearly, the Berry phases mentioned above are calculated using the gauge dependent Berry connection. However, since the final result was gauge independent, there should be a gauge independent way to calculate it. This is provided by the Berry curvature that plays the role of a magnetic field in the parameter space and is a more fundamental quantity than the Berry connection. The example of spin-1/2 particle in a magnetic field is often used to demonstrate the basic concepts and important properties of the Berry phase [2,7,8]. The corresponding curvature in its vector form is an effective magnetic field in parameter space generated by a Dirac magnetic monopole of strength $-1/2$ in the origin as shown in Fig. 1(a), where we present the cross section for the field distribution on the surface of a unit sphere. Obviously, it is homogeneous and isotropic in parameter space. Berry phase can then be interpreted as the magnetic flux through the area whose boundary is the closed loop. It only depends on the global property of the adiabatic evolutions, which offers potential advantages against local fluctuations for implementing geometric quantum gates [23]. Motivated by this application of Berry phase in phase-shift gate operation in quantum computation [3–6], we investigate the effect of the geometric phase of the Rabi model from the viewpoint of geometric connection and curvature.

In this paper we calculate the geometric phase of the two-qubit Rabi model where two qubits interact with the quantized field inhomogeneously. In Section 2 we first review the analytical results for eigensolutions of the two-qubit Rabi model with the RWA according to the conservation of total number of excitation and present the analytical expressions of the Berry phase for the eigenstates. The procedure is as follows: From the connection we derive the curvature, the integration of which gives the phase. Then we turn to the geometric curvature and phase for noneigenstates in order to tackle the pure geometric effect induced by the vacuum photon state. The system returns back to its initial state after a period, which realizes a vacuum-to-vacuum cyclic evolution. We analyze the corresponding geometric curvature field, in comparison with the example of spin-1/2 particle. In Section 3 we study the Berry phase of the two-qubit Rabi model beyond the RWA. The adiabatic approximation is adopted to derive the analytical results for Berry phases of the eigenstates. For strong coupling case we truncate the Hilbert space and calculate the phases numerically. The geometric phases for the evolution of noneigenstates are compared with and without the RWA. Section 4 summarizes our main findings.

2. Geometric phase under the RWA

2.1. Berry phase for eigenstates

The two-qubit Rabi model is described by the Hamiltonian ($\hbar = 1$) [20]

$$H = \omega_c a^\dagger a + \sum_{j=1,2} \left(\frac{\omega_j}{2} \sigma_j^z + g_j \left(a^\dagger + a \right) \sigma_j^x \right). \tag{1}$$

Here $a^\dagger$ (a) is the bosonic creation (annihilation) operator of field mode with frequency ω_c, and ω_j denotes the energy splitting of qubit j described by Pauli matrices σ_j^x, σ_j^y and σ_j^z. We notice that the