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# Analytic solutions in the dyon black hole with a cosmic string: Scalar fields, Hawking radiation and energy flux

H.S. Vieira [a,b,*], V.B. Bezerra [a], G.V. Silva [a]

[a] *Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, CEP 58051-970, João Pessoa, PB, Brazil*

[b] *Centro de Ciências, Tecnologia e Saúde, Universidade Estadual da Paraíba, CEP 58233-000, Araruna, PB, Brazil*

## HIGHLIGHTS

- A cosmic string is introduced along the axis of symmetry of the dyonic black hole.
- The covariant Klein–Gordon equation for a charged massive scalar field in this background is analyzed.
- Both angular and radial parts are transformed to a confluent Heun equation.
- The resulting Hawking radiation spectrum and the energy flux are obtained.

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## ABSTRACT

Charged massive scalar fields are considered in the gravitational and electromagnetic field produced by a dyonic black hole with a cosmic string along its axis of symmetry. Exact solutions of both angular and radial parts of the covariant Klein–Gordon equation in this background are obtained, and are given in terms of the confluent Heun functions. The role of the presence of the cosmic string in these solutions is showed up. From the radial solution, we obtain the exact wave solutions near the exterior horizon of the

* Corresponding author at: Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, CEP 58051-970, João Pessoa, PB, Brazil.

*E-mail addresses:* horacio.santana.vieira@hotmail.com (H.S. Vieira), valdir@fisica.ufpb.br (V.B. Bezerra), gislainevs@hotmail.com (G.V. Silva).

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black hole, and discuss the Hawking radiation spectrum and the energy flux.



## 1. Introduction

The knowledge of the behavior of different fields which interact with the gravitational field of black holes, can give us, in principle, some relevant informations about the physics of these objects. In particular, the scalar field constitutes one of these fields whose behavior will be studied in this paper taking into account a background spacetime generated by a rotating black hole with electric and magnetic charges through which passes a cosmic string. Along this line of research, the separability of the Klein–Gordon equation has been studied in different black hole backgrounds [1–3]. Other studies concerning the behavior of scalar fields in different black holes background as well as their consequences can be found in [4–6].

The topological defect called cosmic string is predicted in some gauge field theories [7] as a result of phase transitions. It can either form closed loops or extend to infinity, and is characterized by its tension, $G\mu$, where $G$ is the Newton's gravitational constant and $\mu$ is the mass per unit length of the string. The spacetime geometry associated with a straight infinite and infinitely thin cosmic string has a conical structure which means that it is locally flat but not globally, due to the fact that on the localization of the string the curvature tensor has a delta shaped form. The local flatness the spacetime surrounding a cosmic string means that there is no local gravitational force. The section perpendicular to the cosmic string has an azimuthal deficit angle given by $\Delta\phi = 8\pi G\mu$ [8]. However, there exist some interesting gravitational effects associated with the global features of the spacelike section around the cosmic string. Among these effects, a cosmic string can induce a finite electrostatic self-force on an electric charged particle, it can induce the emission of radiation by a freely moving particle, act as a gravitational lens among others [9].

A cosmic string cannot appear, necessarily, as a single object in empty space. In fact, it can appear as part of a larger gravitational system, as for example, a black hole. In which case the cosmic string is included [10] by removing a wedge, that is, by requiring that the azimuthal angle around the axis of symmetry runs over the range $0 < \phi < 2\pi b$, with $b = 1 - 4\mu$. Then, gluing together the resulting edges we get the spacetime corresponding to a black hole with a cosmic string passing through it. As an example of such generalizations we can mention the Schwarzschild spacetime with a cosmic string [11] and the Kerr spacetime with a cosmic string [12].

An interesting phenomenon which corresponds to a spontaneous emission of black body radiation by black holes is the Hawking radiation, which was predicted from the study of the thermal radiation emitted by a spherically symmetric black hole [13]. The studies concerning this phenomenon was carried on by using different methods [14–16]. In particular, the emission of scalar particles by black holes has been an object of intense investigations in recent years [17,18].

Zhao and Zhang [19] by introducing the tortoise coordinate to discuss the Hawking thermal spectrum from Kerr–Newman–Kasuya black hole. Other approaches include the investigation of the general radiation spectrum by using the tunneling mechanism process adopted by Yang [20] and the one which consider the Hawking radiation using the viewpoint of quantum anomalies considered by He et al. [21]. In the methods used by these authors it is not necessary the full solution of the scalar equation.

Our contribution is obtaining the exact solutions of the Klein–Gordon equation for a charged massive scalar field in the Kerr–Newman–Kasuya spacetime (dyonic black hole) with a cosmic string passing through it. These solutions are given in terms of the confluent Heun functions [22] and are valid in the region between the exterior event horizon and infinity, which corresponds to the whole space. If we cancel the magnetic charge, namely, take $Q_m = 0$, we particularize the solutions to the