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Invariant currents and scattering off locally symmetric potential landscapes

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HIGHLIGHTS

- We show that local discrete symmetries yield invariant currents.
- Bloch and parity theorems are generalized when the associated symmetries are broken.
- Formulation of scattering via the symmetry-induced invariant currents.
- We provide sum rules for the invariant currents characterizing perfect transmission.

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ABSTRACT

We study the effect of discrete symmetry breaking in inhomogeneous scattering media within the framework of generic wave propagation. Our focus is on one-dimensional scattering potentials exhibiting local symmetries. We find a class of spatially invariant nonlocal currents, emerging when the corresponding generalized potential exhibits symmetries in arbitrary spatial domains. These invariants characterize the wave propagation and provide a spatial mapping of the wave function between any symmetry related domains. This generalizes the Bloch and parity theorems for broken reflection and translational symmetries, respectively. Their non-vanishing values indicate the symmetry breaking, whereas a zero value denotes the restoration of the global symmetry where the well-known forms of the two theorems are recovered. These invariants allow for a systematic treatment of systems with any local

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symmetry combination, providing a tool for the investigation of the scattering properties of aperiodic but locally symmetric systems. To this aim we express the transfer matrix of a locally symmetric potential unit via the corresponding invariants and derive quantities characterizing the complete scattering device which serve as key elements for the investigation of transmission spectra and particularly of perfect transmission resonances.

1. Introduction

Symmetries constitute one of the cornerstones of physics, being prominently displayed due to their fundamental role in the theoretical treatment of any system. Symmetry principles not only reduce the extent of information which is required for the description of a physical system but also dictate the form of physical laws. Under this prism, the endeavor to extend the frontiers of our knowledge about nature is significantly based on the discovery of higher symmetry principles.

The usual pathway is to consider symmetry principles which hold *globally* for a physical system, explaining important phenomenological properties and facilitating the mathematical description. However, global symmetry usually is an idealized scenario, mainly met in models, approximative schemes or structurally simple isolated systems. The concept of local symmetry is usually introduced in the context of gauge transformations involving space–time dependent parameters which, in turn, imply that the associated symmetry is valid at a single space–time point.

Between these two symmetry classes, the one valid at every point of space (or space–time) and the other valid at a single point, another category can be defined, where different symmetries are fulfilled in different spatial domains of finite extent. Physical systems possessing the latter property usually emerge due to the breaking of a global symmetry so that new symmetries at different scales occur. Such symmetry-associated patterns are manifest in extensively diverse structures encountered in nature [1] and dominate several length scales. Therefore, it is a generic situation to deal with extended physical systems involving domains which are locally characterized by a certain symmetry.

Spatially localized symmetries can be intrinsic in complex systems such as, e.g., large molecules [2–4], quasicrystals [5–8], self-organized, pattern-forming systems [9] or partially disordered matter [10]. On the other hand, they can be present by design in multilayered photonic devices [11–16], quantum semiconductor superlattices [17], acoustic waveguides [18] or magnonic systems [19]. Moreover, technological advances often require the breaking of global discrete symmetries in order to obtain flexible structures suitable for applications.

Despite the fact that systems belonging to the aforementioned classes have been extensively investigated, the theoretical framework for their mathematical description is usually restricted to the case of global symmetries. On a local level, studies on the structural features which affect spectral and localization properties has been carried out [20], for instance in hybrid systems which are comprised of domains each with different quasiperiodic structure. However, little attention has been paid to the impact of explicit *local symmetries*, implied by the breaking of a global symmetry, although they are obviously present and often coexist at different spatial scales. A rigorous theoretical treatment which addresses the local symmetry induced properties is missing and obviously a new point of view needs to be introduced.

A step towards this direction was recently made by introducing the concept of *local parity* (LP) [21]. It was shown how a system's decomposition into mirror symmetric units relates to spectral properties and especially to perfect transmission resonances (PTRs) in aperiodic setups. The origin of perfect transmission in such systems, even though tentatively linked to symmetry concepts [22,23], had yet been an unresolved issue, lacking a rigorous theory which directly relates perfect transmission in aperiodic and quasiperiodic setups to its underlying (local) symmetry properties. Within the