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Many-body localization in disorder-free systems: The importance of finite-size constraints

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ABSTRACT

Recently it has been suggested that many-body localization (MBL) can occur in translation-invariant systems, and candidate 1D models have been proposed. We find that such models, in contrast to MBL systems with quenched disorder, typically exhibit much more severe finite-size effects due to the presence of two or more vastly different energy scales. In a finite system, this can artificially split the density of states (DOS) into bands separated by large gaps. We argue for such models to faithfully represent the thermodynamic limit behavior, the ratio of relevant coupling must exceed a certain system-size depedent cutoff, chosen such that various bands in the DOS overlap one another. Setting the parameters this way to minimize finite-size effects, we study several translation-invariant MBL candidate models using exact diagonalization. Based on diagnostics including entanglement and local observables, we observe thermal (ergodic), rather than MBL-like behavior. Our results suggest that MBL in translation-invariant systems with two or more very different energy scales is less robust than perturbative arguments suggest, possibly pointing to the importance of non-perturbative effects which induce delocalization in the thermodynamic limit.

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1. Introduction

One of the remarkable consequences of quantum mechanics is the localization of a single particle moving in a disordered medium in one and two spatial dimensions [1]. This phenomenon, known as Anderson localization, relies on the presence of a static "quenched" disorder landscape through which a particle can hop, causing its eigenstates to form localized wave packets instead of extended (Bloch) states in a usual crystal [see Fig. 1(a)]. This has dramatic consequences for dynamics and transport [2].

Recently it was found that localization can persist in interacting systems of many particles [3–6]. One of the simple examples of such systems is shown in Fig. 1(b); it consists of a 1D lattice half-filled with particles that can hop as well as interact between nearest neighbour sites. Moreover, similar to the Anderson case, the system is in the background of an external disorder potential that is considered static and only enters the description as a random chemical potential on each site. Recent work [3–6] has shown that for sufficiently strong disorder such a system enters a "many-body localized" (MBL) phase where, similar to the Anderson case, transport is inhibited even at non-zero temperature. While in the Anderson case the localization directly implies spatial locality of the wave function of typical eigenstates, in the interacting case each eigenstate $|\Psi\rangle$ is a linear combination of Fock states $|n_1, n_2, \ldots, n_L\rangle$, where n_i is the number operator on site i. There are exponentially many possible $|\{n_i\}\rangle$ and in a thermal (delocalized) system, $|\Psi\rangle$ has a non-zero component on almost all of them. When disorder is strong enough, it turns out that a typical $|\Psi\rangle$ has non-zero projection on a far fewer $|\{n_i\}\rangle$. This is a signature of many-body localization, sometimes referred to as "localization in Fock space" because of the system's inability to explore the entire many-body Hilbert space.

A special structure of eigenstates $|\Psi\rangle$ in the MBL phase generally is a result of an emergence of an extensive number of locally conserved quantities [7–11]. The appearance of these local integrals of motion is responsible for the ergodicity breaking in the MBL phase, and leads to a characteristic entanglement structure [7,12,13] of its individual many-body eigenstates. In ergodic (thermalizing) systems, the entanglement of typical eigenstates at high energy densities above the ground state obeys the so-called "volume law": the von Neumann entropy of a large finite subsystem in such an eigenstate scales as the total number of degrees of freedom in the subsystem. This is simply a manifestation of $|\Psi\rangle$ being a random linear combination of nearly all Fock states $|\{n_i\}\rangle$. In contrast, the extensive number of local integrals of motion (which span the entire many-body Hilbert space) constrains the eigenstates in the MBL phase to have significantly "less" entanglement: their von Neumann entropy only scales as the *area* of the subsystem (therefore, in 1D the entanglement entropy is a constant). Further, dynamical probes such as the spreading of entanglement entropy following a global quench [14–16], as well as the time evolution of local observables [17–19], show that MBL phases have universal properties that distinguish them from both the ergodic phase and the non-interacting Anderson insulator.

One of the outstanding questions is whether the MBL phase necessarily requires disorder, i.e. could MBL-like physics and ergodicity breaking arise in translation-invariant systems, solely as a consequence of interactions? Recent works suggest that this intriguing phenomenon may indeed be possible [20–31]. A generic class of proposed translation-invariant MBL models involves two species of fermions a and b hopping on a 1D lattice

$$H = -J\sum_i a_i^\dagger a_{i+1} - \lambda \sum_i b_i^\dagger b_{i+1} + h.c. + \sum_{k,l} U(k-l) a_k^\dagger a_{k+\sigma} b_l^\dagger b_{l+\sigma'}. \tag{1}$$

Here J represents the hopping amplitude of fast (a) particles and $\lambda \ll J$ is the hopping of heavy (b) particles. The particles also interact via $U(k-l)$. In the simplest case, $\sigma = \sigma' = 0, k = l$ and the interaction reduces to an on-site density–density term, thus the model is formally identical to the 1D Hubbard model [31]. This model is schematically shown in Fig. 1(c) and can be viewed as a generalization of MBL models when the disorder is no longer static. That is, the particles that generate the disorder are included in the system and allowed to undergo their own quantum dynamics and interact with the original particles of the system. Apart from the 1D Hubbard model, we will also consider an alternative model with $\sigma = 1, \sigma' = 0, k = l$ which can be visualized as light particles hopping on a lattice subject to the kinematic constraint depending on whether the hop extends across a heavy particle ("barrier") or not [23]. Finally, we will also study a related one-component model