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Adiabatic Landau–Zener transitions at avoided level crossings with fast noise

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ABSTRACT

Effects of a fast classical noise on *adiabatic* Landau–Zener (LZ) transitions between the $(2S + 1)$ Zeeman multiplets (diabatic states) of an arbitrary spin S at an *avoided* level crossing are investigated. The spin system is simultaneously coupled to a *slow* regular magnetic field and a *fast* random field with Gaussian realizations. In the longitudinal direction, the magnetic field changes its sign at the degeneracy point (and is unbounded at large positive and negative times $t = \pm\infty$ far from the degeneracy point) while in its single transverse direction, it remains of constant amplitude. The noise is considered in the limit where its characteristic correlation time (decay time) is small enough compared to the characteristic time of adiabatic LZ transitions. With these considerations, the condition for adiabatic evolution allows us to analytically evaluate the populations of diabatic levels and coherence factors. The study is first implemented for two- $(S = 1/2)$ and three- $(S = 1)$ state systems and finally extended to arbitrary S. A numerical study is implemented allowing us to check/confirm the range of validity of our analytical solutions. We found a satisfactory quantitative agreement between numerical and analytical data.

1. Introduction

As is well known, the adiabatic passage technique was first implemented using nuclear magnetic resonance systems [1]. In these systems, by slowly varying the magnetic field frequency or direction,

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the population transfer between nuclear spin states was achieved with a relatively good efficiency. Since then, the technique has attracted tremendous interest and has been used for coherent population transfer in many fields, ranging from optical transitions in atomic and molecular systems to trapped singly ionized alkaline-earth ions, or more generally from chemistry to nuclear physics [1–5]. Naturally, the technique is based on one of the most powerful concepts in quantum physics: the adiabatic theorem which asserts that a quantum system slowly sweeps across an avoided level crossing remains in its instantaneous eigenstate if its coupling with other states is less than the energy separations between them [6].

However, when passing through an avoided level crossing with a slowly varying dynamical parameter, there are systems for which adiabaticity principles break down, resulting in nonadiabatic transitions (transitions between adiabatic states). This is achieved when two quantum levels come close by linear variation of a controlling parameter. The first of such observations was formulated and discussed independently by Landau [7], Zener [8], Stückelberg [9] and Majorana [10] in what is today known as the Landau–Zener (LZ) model. In general, this model describes two quantum isolated states that come close in the course of a linearly sweeping external transverse field of constant amplitude b_x and a time-dependent longitudinal field $\dot{b}_z t$ (with $\dot{b}_z$ being the constant sweep velocity; the dot on functions denotes time derivative) that passes through resonance with the transition frequency [11]. Two complementary regimes of driving are generally of interest: the rapid and slow driving regimes [12]. In the rapid driving regime (non-adiabatic evolution), the two levels (diabatic states) cross systematically at $t = 0$. The system rapidly traverses this region with a transition time estimated as $1/\sqrt{\dot{b}_z}$ [13,14] with a probability $P_{LZ}(\infty) = \exp[-2\pi b_x^2/\dot{b}_z]$ to stick in the same diabatic state at time $t = \infty$ [7–10]. For slow driving (adiabatic evolution), the diabatic states hybridize around the crossing region creating an avoided crossing [7–10]. The system accumulates a phase [9] that gives rise to periodic variations in the populations [9]. The instantaneous eigenstates ensure population transfer within a duration estimated as $b_x/\dot{b}_z$ [13,14] between two different diabatic states with a probability of $1 - P_{LZ}(\infty)$. Thus, the LZS process consists of both a non-adiabatic level transition and an adiabatic phase accumulation [7–10].

Moreover, spin systems are in general found in host lattices (real or artificial) where they are not immunized to environmental effects [15–18]. For applications in quantum computing, the systems should preserve their coherent states during a time longer than decoherence time [19–21]. Unfortunately, this is not the case in realistic situations (quantum dots, quantum wells, etc.). Nuclear spin interactions create a magnetic field random in magnitude and directions called the Overhauser's field associated with hyperfine interactions [18,20,19]. These interactions are real sources of decoherence (rapid spin dephasing) and open a splitting (avoided crossing) at degeneracy point [18,20,19] (see Fig. 1). For these reasons, it is important to analyze the effects of hyperfine interactions on spins at avoided level crossing. Quantum mechanically, the Overhauser field is a fluctuating spin bath. We assume that nuclear states in the bath slightly change after a linear sweep. Thus, we treat the Overhauser field classically as a random magnetic field whose statistical distribution reflects the quantum fluctuations of the magnetic ensemble(the bath). At operating temperature (normal experimental conditions), the thermal energy is exceedingly larger than the nuclear energy due to the external Zeeman field, the nuclei spins are completely unpolarized [20,19]. This results in a Gaussian distribution of nuclear fields [20,19]. Thence, when in addition the nuclear-spin coupling strength is weak enough and the relaxation of the nuclear bath is fast, nuclear dynamics effects are treated as a fast Gaussian noise [15–18]. In other words, the noise is fast when the nuclear relaxation time T_{nuc} is sufficiently smaller than the characteristic time t_{Zee} of the external Zeeman field ($T_{nuc} \ll t_{\mathrm{Zee}}$). In the opposite situation (slow classical Gaussian noise), Zeeman effects prevail on thermal effects, all spins in the bath are polarized (the nuclear relaxation time $T_{nuc} \gg t_{\mathrm{Zee}}$). The nuclear spins evolve more slowly than electron spins and the Overhauser field is quasi-static over sufficiently short time intervals [19]. As far as this paper is concerned, we will not discuss the case of slow noise.

Furthermore, noise originates from various sources (not only hyperfine interactions) among which should be noted, shot noise generated from metal wires [22], generation–recombination noise [23], doped semiconductors [24], thermal noise [25], etc. For the sake of generalization, we analyze the