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Explicit mass renormalization and consistent derivation of radiative response of classical electron

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A R T I C L E I N F O

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A B S T R A C T

The radiative response of the classical electron is commonly described by the Lorentz–Abraham–Dirac (LAD) equation. Dirac's derivation of this equation is based on energy and momentum conservation laws and on regularization of the field singularities and infinite energies of the point charge by subtraction of certain quantities: "We... shall try to get over difficulties associated with the infinite energy of the process by a process of direct omission or subtraction of unwanted terms". To substantiate Dirac's approach and clarify the mass renormalization, we introduce the point charge as a limit of extended charges contracting to a point; the fulfillment of conservation laws follows from the relativistic covariant Lagrangian formulation of the problem. We derive the relativistic point charge dynamics described by the LAD equation from the extended charge dynamics in a localization limit by a method which can be viewed as a refinement of Dirac's approach in the spirit of Ehrenfest theorem. The model exhibits the mass renormalization as the cancellation of Coulomb energy with the Poincaré cohesive energy. The value of the renormalized mass is not postulated as an arbitrary constant, but is explicitly calculated. The analysis demonstrates that the local energy–momentum conservation laws yield dynamics of a point charge which involves three constants: mass, charge and radiative response coefficient θ. The value of θ depends on the composition of the adjacent potential which generates Poincaré forces. The classical value of the radiative response coefficient is singled out by the global requirement that the adjacent potential does not affect the radiated energy

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balance and affects only the local energy balance involved in the renormalization.

1. Introduction

The dynamics of the classical electron which involves its radiative response is captured by a certain subset of solutions to the Lorentz–Abraham–Dirac (LAD) equation which has the form, [1–6]:

$$m\dot{v}_{\mu} = ev_{\nu}F^{\nu}_{\mathrm{ex}\mu} + \frac{2}{3}e^2\ddot{v}_{\mu} + \frac{2}{3}e^2\left(\dot{v}\dot{v}\right)v_{\mu}. \tag{1}$$

Here the covariant notation is used, v_μ is 4-velocity of a charged point, $f_{\mathrm{ex}\mu} = ev_\nu F^{\nu}_{\mathrm{ex}\mu}$ is the Lorentz force generated by the external field $F^{\mu\nu}_{\mathrm{ex}}$, $\dot{v}_\mu = \partial_s v_\mu$ and $(\cdot\cdot)$ is the 4-product in Minkowski space:

$$(vw) = v_\nu w^\nu = g_{\mu\nu}v_\mu w_\nu = v_0w_0 - v_1w_1 - v_2w_2 - v_3w_3 = v_0w_0 - \mathbf{v}\cdot\mathbf{w}; \tag{2}$$

we use the units in which the speed of light $\mathrm{c} = 1$ and the summation convention. The LAD equation does not describe the magnetic moment of the electron, for generalizations in this direction see [7,8]. The radiative response of an electron was originally derived by Abraham and Lorentz from the analysis of the Lorentz–Abraham model, see [5,9–12]. A relativistic treatment of the Lorentz–Abraham model meets with difficulties, [2,4,5,9–13]. The relativistic derivation of the radiative response based on the analysis of the energy and momentum conservation laws and on mass renormalization is due to Dirac [1] and now is often used, [2–4]. It is well-known that the derivation of the LAD equation and the equation itself is not without difficulties, [2,5,6,12]. Sometimes the source of the difficulties is attributed to the mass renormalization. The mass renormalization is described in [3, Sec. 75] as follows: "When in the equation of motion we write a finite mass for the charge, then in doing this we essentially assign to it a formally infinite negative "intrinsic mass" of nonelectromagnetic origin, which together with the electromagnetic mass should result in a finite mass for the particle. Since, however, the subtraction of one infinity from another is not an entirely correct mathematical operation, this leads to a series of further difficulties". One could think that since a constant can be added to the total energy, then the infinite self-energy is not a problem. But one has to fulfill the mass renormalization in dynamical regimes with acceleration, where external forces may change the energy of the charge, and it cannot be considered constant. In addition, in a relativistic setting one cannot treat the energy as an independent quantity and must consider the energy–momentum 4-vector. Therefore, it is not clear that the mass renormalization in non-trivial dynamical regimes with self-interaction is "an entirely correct mathematical operation". An implicit assumption that the removal of infinities from a model is a "surgery" which results only in a change of the renormalized parameter of the model and does not have any side effects is not necessarily true. But we show below that though side effects exist, they are controllable.

A natural way to deal with the difficulties in the treatment of infinities is to introduce an extended charge and then apply to it Dirac's analysis to find the limit dynamics in the point localization limit as its size tends to zero. But there are difficulties in this approach. Namely, [4, Sec. 8.4]: "Unfortunately, it is not easy to obtain a theory in this way that has a local conservation of energy and momentum". Nevertheless, we follow this path; a different approach to the dynamics of a point charge in EM field, which does not use renormalization and is based on Maxwell–Born–Infeld equations, is developed in [14]. The fulfillment of the local energy–momentum conservation laws up to the point limit constitutes an important part of Dirac's method: "The usual derivation of the stress-tensor is valid only for continuous charge distributions and we are here using it for point charges. This involves adopting as a fundamental assumption the point of view that energy and momentum are localized in the field in accordance with Maxwell's and Pointing's ideas", [1, p. 152].