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An angular frequency dependence on the Aharonov–Casher geometric phase

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ABSTRACT

A quantum effect characterized by a dependence of the angular frequency associated with the confinement of a neutral particle to a quantum ring on the quantum numbers of the system and the Aharonov–Casher geometric phase is discussed. Then, it is shown that persistent spin currents can arise in a two-dimensional quantum ring in the presence of a Coulomb-type potential. A particular contribution to the persistent spin currents arises from the dependence of the angular frequency on the geometric quantum phase.

1. Introduction

In recent decades, the Aharonov–Casher effect [1] has attracted a great deal of attention. This quantum effect arises in the quantum dynamics of a neutral particle with a permanent magnetic dipole moment and consists in a phase shift in the wave function of the neutral particle [2,3]. This phase shift arises from the interaction between the permanent magnetic dipole moment of the neutral particle and an electric field, and it is given by

$$\phi_{\text{AC}} = \oint \left(\vec{\mu} \times \vec{E}\right) \cdot d\vec{r} = \pm 2\pi\mu\lambda, \tag{1}$$

where μ is the magnetic dipole moment and λ is a constant associated with a linear electric charge density. Discussions about the Aharonov–Casher effect have been extended to the nondispersivity and nonlocality [4], and also to the topological nature [5]. The Aharonov–Casher effect [1] has

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also been investigated in recent years in the Lorentz-symmetry violation background [6,7], in the noncommutative quantum mechanics [8], in noninertial reference frames [9–12], in holonomic quantum computation [13] and with respect to its dual effect, which is called in the literature as the He–McKellar–Wilkens effect [14]. The gravitational analogue of the Aharonov–Casher effect was obtained in Ref. [15].

Geometric quantum phases [16,17] have also been studied in mesoscopic systems where there exists a dependence of the energy levels on the geometric phase [18]. This dependence on the geometric phase gives rise to a quantum effect called as the Aharonov–Bohm effect for bound states [19,20]. Moreover, this flux dependence of the energy levels yields the appearance of persistent currents in mesoscopic systems [18,21–23]. Based on a neutral particle system that possesses a permanent magnetic dipole moment, then, persistent spin currents have been studied in Refs. [24,25].

In this paper, persistent spin currents in a two-dimensional quantum ring are investigated. It is shown that bound states can be obtained for a neutral particle possessing a permanent magnetic dipole moment confined to a two-dimensional quantum ring in the presence of a Coulomb-type potential. By considering the interaction between the permanent magnetic dipole moment of the neutral particle and a radial electric field, then, three quantum effects can be observed in this system: one is the dependence of the energy levels on the Aharonov–Casher geometric phase; the second quantum effect is a dependence of the angular frequency associated with the confinement of a neutral particle to a quantum ring on the quantum numbers of the system and also on the geometric quantum phase; the third quantum effect is the arising of persistent spin currents in the two-dimensional quantum ring. A particular contribution to the persistent spin currents comes from the dependence of the angular frequency on the geometric quantum phase. It is worth mentioning that this last contribution has not been reported in the literature yet thus, as an example, we obtain both angular frequency and the persistent spin current associated with the ground state of the system.

The structure of this paper is as follows: in Section 2, we discuss the quantum dynamics of neutral particle that possesses a permanent magnetic dipole moment confined to a two-dimensional quantum ring in the presence of a Coulomb-type potential. We show that the energy levels depend on the Aharonov–Casher geometric phase, and also show a dependence of the angular frequency on the quantum numbers of the system and on the geometric quantum phase. Finally, we obtain both angular frequency and the persistent spin current associated with the ground state of the system; in Section 3, we present our conclusions.

2. Quantum dynamics of a spin-half neutral particle

Let us start with the relativistic quantum dynamics of a neutral particle with permanent magnetic dipole moment that interacts with external magnetic and electric fields. This quantum dynamics can be described by introducing a nonminimal coupling into the Dirac equation as $i\gamma^{\mu}\partial_{\mu} \rightarrow i\gamma^{\mu}\partial_{\mu} + \frac{\mu}{2}F_{\mu\nu}(x)\,\Sigma^{\mu\nu}$, where $F_{\mu\nu}(x)$ is the electromagnetic tensor, with $F_{0i} = E_i$, $F_{ij} = -\epsilon_{ijk}B^k$ and μ is the permanent magnetic dipole moment of the neutral particle [1–3]. In particular, in the Aharonov–Casher system [1], the magnetic moment of the neutral particle interacts with an electric field produced by a linear distribution of electric charges, $\vec{E} = \frac{\lambda}{\rho}\hat{\rho}$, where $\rho = \sqrt{x^2+y^2}$, and λ is a constant associated with the linear charge distribution along the z axis. Based on this symmetry, from now on, we need to deal with the Dirac equation in cylindrical coordinates (curvilinear coordinates). The mathematical formulation used to write the Dirac equation in curvilinear coordinates is the same of spinors in curved spacetime [26–28]. Then, in cylindrical coordinates, the line element of the Minkowski spacetime is writing in the form: $ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2$ ($\hbar = c = 1$). Therefore, by applying a coordinate transformation $\frac{\partial}{\partial x^{\mu}} = \frac{\partial \bar{x}^{\nu}}{\partial x^{\mu}}\frac{\partial}{\partial \bar{x}^{\nu}}$ and a unitary transformation on the wave function $\psi(x) = U\,\psi'(\bar{x})$, the Dirac equation for a neutral particle with permanent magnetic dipole moment can be written in any orthogonal system in the following form [7,28]: $i\,\gamma^{\mu}D_{\mu}\Psi + \frac{i}{2}\sum_{k=1}^{3}\gamma^{k}\left[D_k \ln\left(\frac{h_1 h_2 h_3}{h_k}\right)\right]\Psi + \frac{\mu}{2}F_{\mu\nu}(x)\,\Sigma^{\mu\nu}\Psi = m\Psi$, where $D_{\mu} = \frac{1}{h_{\mu}}\partial_{\mu}$ is the derivative of the corresponding coordinate system and the parameter h_k corresponds to the scale factors of this coordinate system. In our case (cylindrical coordinates), the scale factors are $h_0 = 1$, $h_1 = 1$, $h_2 = \rho$