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An introduction to the physics of Cartan gravity

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ABSTRACT

A distance can be measured by monitoring how much a wheel has rotated when rolled without slipping. This simple idea underlies the mathematics of Cartan geometry. The Cartan-geometric description of gravity consists of a $SO(1,4)$ gauge connection $A^{AB}(x)$ and a gravitational Higgs field $V^A(x)$ which breaks the gauge symmetry. The clear similarity with symmetry-broken Yang–Mills theory suggests strongly the existence of a new field V^A in nature: the gravitational Higgs field. By treating V^A as a genuine dynamical field we arrive at a natural generalization of General Relativity with a wealth of new phenomenology. Importantly, General Relativity is reproduced exactly in the limit that the $SO(1,4)$ norm $V^2(x)$ tends to a positive constant. We show that in regions wherein V^2 varies – but has a definite sign – the Cartan-geometric formulation is a particular version of a scalar–tensor theory (in the sense of gravity being described by a scalar field ϕ, metric tensor $g_{\mu\nu}$, and possibly a torsion tensor $\mathcal{T}_{\mu\nu}{}^{\rho}$). A specific choice of action yields the Peebles–Ratra quintessence model whilst more general actions are shown to exhibit propagation of torsion. Regions where the sign of V^2 changes correspond to a change in signature of the geometry. Specifically, a simple choice of action with FRW symmetry imposed yields, without any additional *ad hoc* assumptions, a classical analogue of the Hartle–Hawking no-boundary proposal with the big bang singularity replaced by signature change. Cosmological solutions from more general actions are described, none of which have a big bang singularity, with most solutions reproducing General Relativity, or its Euclidean version, for late cosmological times. Requiring that gravity couples to matter fields through the gauge prescription forces a fundamental change in the description

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of bosonic matter fields: the equations of motion of all matter fields become first-order partial differential equations with the scalar and Dirac actions taking on structurally similar first-order forms. All matter actions reduce to the standard ones in the limit $V^2 \rightarrow const$. We argue that Cartan geometry may function as a novel platform for inspiring and exploring modified theories of gravity with applications to dark energy, black holes, and early-universe cosmology. We end by listing a set of open problems.

1. Introduction

Riemannian geometry forms the mathematical basis of Einstein's General Relativity. The metric representation of Riemannian geometry consists of the pair of variables $\{g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}\}$. Whilst the metric tensor $g_{\mu\nu}$ encodes all information of distances between points on a manifold, the affine connection $\Gamma^{\rho}_{\mu\nu}$ encodes the information about parallel transport of tangent vectors u^{μ} as well as defining a covariant derivative ∇_{μ} acting on tensors. Within Riemannian geometry the pair $\{g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}\}$ must be *metric-compatible* and *torsion-free*:

- **Metric compatibility:** $\nabla_{\rho} g_{\mu\nu} \equiv \partial_{\rho} g_{\mu\nu} - \Gamma^{\sigma}_{\rho\mu} g_{\sigma\nu} - \Gamma^{\sigma}_{\rho\nu} g_{\mu\sigma} = 0$
- **Zero torsion:** $\Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} = 0$.

The affine connection can then be uniquely determined from the metric

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}) \tag{1}$$

and it becomes natural to view the metric as the primary variable and the affine connection as a secondary, derived quantity.

Despite its monumental success it has long been noted (see e.g. [1,2]) that this description of the gravitational field is quite distinct from that of the force fields of the standard model, i.e. the electroweak and strong forces. The latter two are examples of standard Yang–Mills theories with the electroweak theory being an example of a symmetry-broken gauge theory. On the other hand, gravity in its traditional Riemannian formulation displays only a superficial similarity to a Yang–Mills field (see [3,4] for discussion of the differences). In [5] Weinberg writes:

> ' . . . *I believe that the geometrical approach has driven a wedge between General Relativity and the theory of elementary particles. As long as it could be hoped, as Einstein did hope, that matter would eventually be understood in geometrical terms, it made sense to give Riemannian geometry a primary role in describing the theory of gravitation. But now the passage of time has taught us not to expect that the strong, weak, and electromagnetic interactions can be understood in geometrical terms, and too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics.*'

The aim of this article is to show that a lesser-known formulation of gravity, based on Cartan geometry – whose mathematical ingredients are precisely those of a spontaneously-broken gauge theory – can underpin a more general, alternative theory of gravity that reduces to General Relativity in a specific limit. We shall refer to that formulation as *Cartan gravity* although this name is also frequently used for the Einstein–Cartan formulation of General Relativity [6]. Rather than driving a wedge between gravity and the other forces of the standard model, it describes gravity in the same language as the other forces, i.e. as a Yang–Mills theory. The dynamical fields of Cartan gravity consist of a Yang–Mills