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Criteria for the absence of quantum fluctuations after spontaneous symmetry breaking

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HIGHLIGHTS

- Precise criteria for absence of quantum fluctuations in symmetry-broken states are established.
- It is not sufficient that the order parameter commutes with the Hamiltonian.
- Clarifies relation between quantum fluctuations and type-B Nambu–Goldstone modes.
- Testable through absence of fundamental limit on maximum coherence time of macroscopic superpositions.

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ABSTRACT

The lowest-energy state of a macroscopic system in which symmetry is spontaneously broken, is a very stable wavepacket centered around a spontaneously chosen, classical direction in symmetry space. However, for a Heisenberg ferromagnet the quantum groundstate is exactly the classical groundstate, there are no quantum fluctuations. This coincides with seven exceptional properties of the ferromagnet, including spontaneous time-reversal symmetry breaking, a reduced number of Nambu–Goldstone modes and the absence of a thin spectrum (Anderson tower of states). Recent discoveries of other non-relativistic systems with fewer Nambu–Goldstone modes suggest these specialties apply there as well. I establish precise criteria for the absence of quantum fluctuations and all the other features. In particular, it is not sufficient that the order parameter operator commutes with the Hamiltonian. It leads to a measurably larger coherence time of superpositions in small but macroscopic systems.

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1. Introduction

The Heisenberg ferromagnet has always been an eccentric duckling in the flock of spontaneous symmetry breaking (SSB) states consisting of antiferromagnets, crystals, superfluids, chiral SSB, the Standard Model and many others. This is only exacerbated by being one of the earliest and simplest models demonstrating SSB, used as the archetype in a large portion of the literature. Perhaps because much of its physics can be understood by undergraduate level calculations, have its peculiarities never been put in a larger perspective. Still the subtleties are intricate enough to have sparked debates between the greatest of minds in the past century [1,2].

Why is this state different from all other states? We talk about the following observations, clarified below:

(i) the order parameter operator commutes with the Hamiltonian, is therefore a symmetry generator and is conserved in time;
(ii) two broken symmetry generators correspond to a single, quadratically dispersing Nambu–Goldstone (NG) mode;
(iii) the classical groundstate is an exact eigenstate of the Hamiltonian, there are no quantum fluctuations;
(iv) the raising operator, a root generator of the symmetry algebra, annihilates the groundstate, even locally (the spin of the maximally polarized state cannot be increased);
(v) there is no *thin spectrum* or *tower of states* of nearly vanishing energy just above the groundstate;
(vi) the groundstate is an eigenstate of the unbroken symmetry generator with non-zero eigenvalue;
(vii) time-reversal symmetry is spontaneously broken.

Arguably the most important of these features are (i) and (ii): the low-energy spectrum of NG modes is different from what one would expect based on the relativistic Goldstone theorem. This issue had been recognized early on, [3–5], and later generalized [6] to systems other than the ferromagnet, but has basically been solved only in the last ten years or so [7–13]: whenever the order parameter operator Q^k, that obtains a non-zero expectation value $\langle Q^k \rangle$ in the symmetry-broken state, is one of the symmetry generators itself – called a *finite Noether charge density* – then any two spontaneously broken generators Q^i, Q^j that contain this operator in their commutation relation $[Q^i, Q^j] = \sum_k f_{ijk} Q^k$ will in fact excite *the same* NG mode. That mode will have a different, in general quadratic, dispersion relation (a more precise statement will be given below). Therefore (i) implies (ii). For the ferromagnet with magnetization along the z-axis, the spin rotation operator S^z obtains a finite Noether charge density, while S^x and S^y are spontaneously broken and excite the same single spin wave (magnon) with quadratic dispersion. Such modes have been called type-B NG modes, as opposed to the 'regular', linearly dispersing, type-A NG modes.

In a parallel development, several states of matter with broken charge densities and/or quadratically dispersing NG modes other than the ferromagnet have been identified. For instance in spinor Bose–Einstein condensates (BEC) [14], kaon condensates in quantum chromodynamics [7] and Tkachenko modes in superfluid vortices [15]. The question of whether the other special properties of the ferromagnet (iii)–(vii) generalize to such systems arises naturally.

Here I will establish precise criteria for the relations between each of the properties (i)–(vii). I will focus in particular on quantum fluctuations, properties (iii)–(v). When a continuous symmetry is spontaneously broken, there is a continuous manifold of degenerate classical groundstates. In the quantum case, any superposition of these states will be a valid groundstate as well, but tiny external perturbations will favor one particular classical state over all the others. At this point that may seem obvious, but these classical groundstates are almost never eigenstates of the quantum Hamiltonian, which implies unitary time evolution would bring one to a state different from the classical state. This deviation from the classical groundstate is known as *quantum fluctuations* although there is actually no time-dependent behavior, in the same sense as there are no particles and anti-particles "popping into and out of existence" in the QED vacuum. It is perhaps the most striking feature of spontaneous symmetry breaking that in almost all cases the actual quantum groundstate is very close to a classical groundstate [16–18]; for instance the reader's chair is (for all practical purposes) in a