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Higher-order Dirac solitons in binary waveguide arrays

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HIGHLIGHTS

- Higher-order Dirac solitons in nonlinear binary waveguide arrays are numerically demonstrated.
- Amplitude profiles of higher-order Dirac solitons are periodic during propagation.
- The period of higher-order Dirac solitons decreases when the soliton order increases.

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ABSTRACT

We study optical analogues of higher-order Dirac solitons (HODSs) in binary waveguide arrays. Like higher-order solitons obtained from the well-known nonlinear Schrödinger equation governing the pulse propagation in an optical fiber, these HODSs have amplitude profiles which are numerically shown to be periodic over large propagation distances. At the same time, HODSs possess some unique features. Firstly, the period of a HODS depends on its order parameter. Secondly, the discrete nature in binary waveguide arrays imposes the upper limit on the order parameter of HODSs. Thirdly, the order parameter of HODSs can vary continuously in a certain range.

1. Introduction

Waveguide arrays (WAs) have been used intensively to simulate the evolution of a non-relativistic quantum mechanical particle in a periodic potential [1]. Many fundamental phenomena in

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non-relativistic classical and quantum mechanics such as Bloch oscillations [2,3] and Zener tunneling [4,5] have been investigated both theoretically and experimentally by using WAs. It was shown in recent studies that most of nonlinear phenomena usually associated to fiber optics (such as the emission of resonant radiation from solitons and soliton self-wavenumber shift) can also take place in specially excited WAs, but in the spatial domain rather than in the temporal domain [6,7]; and the supercontinuum in both frequency and wave number domains can be generated in nonlinear WAs [8]. Binary waveguide arrays (BWAs) have been used to mimic relativistic phenomena typical of quantum field theory, such as Klein tunneling [9,10], *Zitterbewegung* (trembling motion of a free Dirac electron) [11,12], and fermion pair production [13], which are all based on the properties of the Dirac equation [14]. The discrete gap solitons in BWAs in the *classical* context have been investigated both numerically [15–17] and experimentally [18]. Gap and out-gap solitons and breathers in BWAs have been investigated in great detail both analytically and numerically [19–21]. These gap solitons were already known in [22] in 1992 for diatomic lattices, and later explicitly derived (in their exact continuum-limit form) for the BWA system in [19] in 2011. Recently, the explicit suggestion to use BWAs to simulate a quantum nonlinear Dirac equation has been put forward in [23] where the gap solitons in BWAs have been shown to be connected to Dirac solitons (DSs) in a nonlinear extension of the relativistic one-dimensional (1D) Dirac equation describing the dynamics of a freely moving relativistic particle. Other soliton solutions have been found for the nonlinear 1D Dirac equation [24], but with a different kind of nonlinearity, in the context of quantum field theory. The 1D DS stability, its dynamics and different scenarios of soliton interaction have been systematically investigated in [25]. The formation and dynamics of two-dimensional DSs in square binary waveguide lattices have been investigated in [26]. Although there is currently no evidence for fundamental quantum nonlinearities, nonlinear versions of the Dirac equation have been studied since a long time. One of the earlier extensions was investigated by Heisenberg [27] in the context of field theory and was motivated by the question of mass. In the quantum mechanical context, nonlinear Dirac equations have been used as effective theories in atomic, nuclear and gravitational physics [28–31]. To this regard, BWAs can offer a unique platform to simulate nonlinear extensions of the Dirac equation when probed at high light intensities. One of these possibilities is to use BWAs as a classical simulator of the Dirac equation to mimic the two-body Dirac model, i.e. the Dirac equation for two interacting relativistic particles, which has attracted interest of researchers since the early days of quantum mechanics [32,33].

Soliton solutions to the well-known nonlinear Schrödinger equation (NLSE) governing the pulse propagation in an optical fiber have been thoroughly investigated among various classes of solitons [34–36]. The shape of the fundamental temporal soliton obtained from the NLSE (further referred to as NLS solitons) is described by the hyperbolic function $u(\tau) = sech(\tau)$. This shape is absolutely retained during propagation of the fundamental NLS soliton along the optical fiber. In light of this, the DS investigated in [23] can also be termed as the fundamental DS, because its profile is also conserved during propagation along the longitudinal axis of BWAs. However, apart from the fundamental soliton solution, the NLSE also has so-called higher-order (HO) soliton solutions with initial shapes being described by $Nsech(\tau)$ where N is an arbitrary integer provided that $N \geq 2$ [35,37,38] (further referred to as HONLS solitons). Unlike the fundamental NLS soliton, HONLS solitons have profiles which repeat periodically during propagation. Thus, it is natural to expect that in addition to the fundamental DS, one can also have HODSs in BWAs whose profiles repeat periodically during propagation. The aim of this work is to investigate the properties and dynamics of these HODSs and to compare them with HONLS solitons. This paves the way for using BWAs to simulate nonlinear extensions of the Dirac equation, as well as other solitonic and non-solitonic effects of nonlinear Dirac equations.

2. Higher-order Dirac solitons

Light propagation in a binary array of Kerr nonlinear waveguides can be described, in the continuous-wave regime, by the following dimensionless coupled-mode equation (CME) [15]:

$$i\frac{da_n(z)}{dz} = -\kappa[a_{n+1}(z) + a_{n-1}(z)] + (-1)^n\sigma a_n - \gamma|a_n(z)|^2 a_n(z), \tag{1}$$