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Entanglement spectrum and block eigenvalue spacing distribution of correlated electron states

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ABSTRACT

Entanglement spectrum of finite-size correlated electron systems are investigated using the Gutzwiller projection technique. The product of largest eigenvalue and rank of the block reduced density matrix, which is a measure of distance of the state from the maximally entangled state of the corresponding rank, is seen to characterise the insulator to metal crossover in the state. The fraction of distinct eigenvalues exhibits a 'chaotic' behaviour in the crossover region, and it shows a 'integrable' behaviour at both insulating and metallic ends. The integrated entanglement spectrum obeys conformal field theory (CFT) prediction at the metal and insulator ends, but shows a noticeable deviation from CFT prediction in the crossover regime, thus it can also track a metal–insulator crossover. A modification of the CFT result for the entanglement spectrum for finite size is proposed which holds in the crossover regime also. The adjacent level spacing distribution of unfolded non-zero eigenvalues for intermediate values of Gutzwiller projection parameter g is the same as that of an ensemble of random matrices obtained by replacing each block of reduced density matrix by a random real symmetric Toeplitz matrix. It is strongly peaked at zero, with an exponential tail proportional to $e^{-(n/R)s}$, where s is the adjacent level spacing, n is number of distinct eigenvalues and R is the rank of the reduced density matrix.

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1. Introduction

Quantum entanglement of a system quantifies the correlations between the parts of the system [1], which serves as a resource for quantum information processing tasks. The block entanglement, viz. the entropy of a subsystem, is a widely-used entanglement measure, that has been used to investigate critical behaviour near quantum phase transitions in spin systems [2,3]. But there are not many studies of the entanglement in interacting electron systems, which exhibit substantially richer structure than interacting spin systems as they carry additional charge degrees of freedom. In this article, we study the strong correlation effect on the entanglement spectrum of the one-dimensional Gutzwiller state [4,5], as a proto-type strongly-correlated state.

The Gutzwiller state:

Gutzwiller state was initially suggested as a variational ground state for the Hubbard model [4]. In this state, the strong on-site correlation effect of the Hubbard model ground state is mimicked by applying a projection operator on the non-interacting metallic state to decrease the double occupancy. At one end of the control parameter is the metallic state with no projection, viz. a Fermi ground state constructed from occupying lowest-lying one-electron plane-wave states for both up and down spin electrons. The metallic state maximises the double occupancy as there is no correlation between the up and down spin electron. At the other extreme, there is an insulator phase, corresponding to the fully-projected state with no double occupancy. The Gutzwiller state for a lattice of N sites is given by

$$|g\rangle = \prod_{i=1}^{N}\{1-(1-g)\hat{n}_{i\uparrow}\hat{n}_{i\downarrow}\}|F\rangle, \tag{1}$$

where $|F\rangle = \prod_{k=0}^{k_F} \hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{k\downarrow}|0\rangle$ is the metallic Fermi state constructed from the vacuum state by using electron creation operators $\hat{c}^{\dagger}_{k\sigma}$ with a momentum k and spin σ, and g is a parameter taking values from 0 to 1, $g = 1$ being the non-interacting case, and $g = 0$ being the limit of infinite interactions. The filling factor is determined by k_F, the Fermi momentum.

While the Gutzwiller state gives good agreement with Hubbard model ground state for three dimensions, in 1D, it is different from the ground state of the Hubbard model. At half-filling, the ground state of the Hubbard model in 1D describes a Luttinger liquid, whereas the Gutzwiller state shows Fermi liquid behaviour [5]. In the thermodynamic limit, for any $g \neq 0$, the momentum space distribution of electrons for the 1D half-filled Gutzwiller state has a discontinuity at the Fermi wave vector. Therefore, in the thermodynamic limit, the system is metallic for any $g \neq 0$. Also, in 1D, the half-filled Gutzwiller state for $g = 0$ is the exact ground state of the Haldane–Shastry model [6], which is a Heisenberg-like model with long range interactions. Hence the Gutzwiller state is, by itself, an interesting correlated electron state. Here we will be interested in the entanglement spectrum of the half-filled Gutzwiller state.

Entanglement entropy, fluctuations and metal–insulator crossover:

The block entanglement entropy and bipartite fluctuations of the Gutzwiller state in 1D for half bipartition has been recently studied as a function of the correlation factor g and the number of sites N [7]. The block entanglement entropy S for half-bipartition of half-filled Gutzwiller state scales as:

$$S = c_{eff}(g, N)\left(\frac{1}{2} + \frac{1}{3}log(N)\right) \tag{2}$$

which has the same form as conformal field theory (CFT) result for one dimensional systems except with an effective central charge $c_{eff}(g, N)$ which is a function of both g and N. At $g = 1, c_{eff} = 2$ and at $g = 0, c_{eff} = 1$ which are independent of N and are the correct results in these two limits from CFT. The N dependence occurs for intermediate values of g. The scaling of $c_{eff}(g, N)$ indicates a metal–insulator crossover. Bipartite fluctuations also show a scaling. The scaling of both bipartite fluctuations and $c_{eff}(g, N)$ show that for $N < 10^5$, the relevant scaling variable is $N^{1/3}g$ and metal–insulator crossover occurs at $N^{1/3}g \approx 0.24$.

The reason behind deviation from the CFT result for intermediate values of g is the existence of a finite correlation length. As shown in Ref. [7], the correlation length between opposite spins is infinite