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Some topological states in one-dimensional cold atomic systems

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ABSTRACT

Ultracold atoms trapped in optical lattices nowadays have been widely used to mimic various models from condensed-matter physics. Recently, many great experimental progresses have been achieved for producing artificial magnetic field and spin–orbit coupling in cold atomic systems, which turn these systems into a new platform for simulating topological states. In this paper, we give a review focusing on quantum simulation of topologically protected soliton modes and topological insulators in one-dimensional cold atomic system. Firstly, the recent achievements towards quantum simulation of one-dimensional models with topological non-trivial states are reviewed, including the celebrated Jackiw–Rebbi model and Su–Schrieffer–Heeger model. Then, we will introduce a dimensional reduction method for systematically constructing high dimensional topological states in lower dimensional models and review its applications on simulating two-dimensional topological insulators in one-dimensional optical superlattices.

1. Introduction

With the discovery of topological insulators, the exploration of exotic topological states has become a forefront research area in condensed-matter physics [1,2]. Topological states of matters are new states of quantum matters that have a bulk energy gap but gapless edge or surface states at their boundaries [1,2]. The classification methods of these states are beyond the traditional way using

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symmetry-breaking order parameters and Landau–Ginzburg theory [3]. Instead, topological states are characterized by topological invariants which are insensitive to the smooth changes of the parameters of the systems and cannot change unless the energy gap closes [1,2]. The tantalizing properties such as robust edge states and exotic non-Abelian excitations allow many applications ranging from semiconductor spintronics [2] to topological quantum computation [4]. Experimentally, topological states have been found in some solid state materials [1,2]; however, the materials with topological features are still quite scarce and the tools to engineer topological states remain to be explored.

On the other hand, ultracold atoms trapped in an optical lattice nowadays have been widely recognized as powerful tools to simulate and study many-body problems originally from condensed matter physics [5,6]. The reason lies in the fact that this system provides clean environment and high controllability, even some extreme physical situations unachieved in condensed matter physics can be reached here. One of the seminal experiments in this regard is the realization of Bose–Hubbard model with ultracold bosonic atoms trapped in an optical lattice, where a quantum phase transition from a superfluid to a Mott insulator has been observed [7,8]. The recent discovery of topological insulators also inspires theorists from cold atom physics to study how to produce artificial gauge fields on neutral atoms [9]. Great experimental progress has been achieved recently along this line, including the generation of artificial magnetic fields (equivalent to Abelian gauge field) in the continuum [10] and in optical lattices [11–13], and also the spin–orbit coupling [14–17] (equivalent to non-Abelian gauge field), which finally leads to the observation of the spin Hall effect in a quantum degenerate Bose gas [18–20]. Therefore, it is expected that cold atoms trapped in an optical lattice could provide a promising platform to realize various topological lattice models and prepare the underlying topological states.

One important hallmark of topological states is the existence of edge states at the interfaces where the topological invariant changes. Historically, the earliest theoretical description of a topological bound state is the one-dimensional (1D) Jackiw–Rebbi model in the context of relativistic field theory [21,22]. This model showed that there would appear a topological zero-energy soliton mode with fractional charge when a fermionic field is coupled to a topologically nontrivial background field. Its counterpart in condensed matter physics is the Su–Schrieffer–Heeger (SSH) model describing zero-energy soliton modes in polyacetylene [23,24]. Actually, the basic physics behind the Jackiw–Rebbi model and the SSH model is governed by a modified Dirac equation. In the past years, there were a lot of theoretical and experimental works using ultracold atoms to simulate the Dirac equation and study the involved physics [25–31], including the experimental observation of Zitterbewegung effect [32,33] and the Klein tunneling [34]. The Dirac Hamiltonian describing the Jackiw–Rebbi model and SSH model has also been engineered with ultracold atomic gas in the continuum [35] and in the optical lattice [36]. The detection of fractional charge number occupying the topological zero mode was extensively investigated [35,37]. Experimentally, the SSH model and its topological feature have recently been realized and probed in a 1D optical superlattice [38].

In two-dimensional (2D) systems in condensed matter physics, the first topological state is the integer quantum Hall state discovered in 1980 [39], which occurs when electrons are confined to two dimensions and subjected to a strong magnetic field perpendicular to the plane. In the bulk sample, electrons occupy Landau levels with a large gap between them. Landau levels can be viewed as bands and the resulted gapped band structure can be classified by a topological invariant called Chern number. In 1988, Haldane has taken one important step towards topological insulators. He introduced the concept of parity anomaly in quantum electrodynamics to construct a quantum Hall state on a honeycomb lattice [40]. This model does not require an external magnetic field but a complex next nearest neighbor hopping. The quantum Hall insulator generated here now is often called as Chern insulator. However, Haldane's model is in the same topological class as the ordinary integer quantum Hall state: 2D system with time-reversal symmetry breaking [41]. At that time, there seemed to be a misconception that topological quantum Hall states can only appear in this situation.

Since the year 2005, it was realized that spin–orbit coupling and time-reversal symmetry could also lead to a topological order called Z_2 topological insulators [42,43]. This discovery directly triggered a lot of theorists and experimentalists beginning to study various symmetry classes of topological phases, including topological insulators and topological superconductors [1,2], which finally brought this trend to the forefront of condensed matter physics. It recently has been generalized to