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Thermofield-bosonization on compact space

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ABSTRACT

We develop the construction of fermionic fields in terms of bosonic ones to describe free and interaction models in the circle, using thermofielddynamics. The description in the case of finite temperature is developed for both normal modes and zero modes. The treatment extends the thermofield-bosonization for periodic space.

1. Introduction

We have discussed quite recently the symmetry aspects of models which describe charge-density waves in incommensurate case within a field theoretic set-up [1]. The bosonization of the fermions [2], has shown to be a most valuable tool. The field theoretic models that describe the phenomenon are constructed with fields in an infinite dimensional space, based upon lower dimensional versions of models used in the description of relativistic elementary particles. In the commensurate case, however, the periodicity of the net of charge distribution and of the sound waves becomes important. Also the finite extent of the sample needs to be addressed. The extension of the field theoretical models which encodes periodic boundary conditions has to be considered in this case. The diversity of new physical phenomena which are described by lower dimensional fermionic field theory models, as for example in graphene and topological insulators, constitute further motivations for this study.

A description of the bosonization of spatially periodic fields has been presented, within a condensed matter context, a long time ago by Haldane [3], and has been the subject of much renewed interest recently [4–6]. However, an approach which, based on field theoretic methods, stresses the

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parallelism with the infinite space case is lacking. We consider here the quantization of scalar and fermionic free fields and the bosonization of fermionic fields in $1+1$ dimensions for spatially periodic boundary conditions. The purpose is to establish which are the essential ingredients that have to be considered in order to account for the space periodicity, what seems to be obscured when the peculiarities of the additional demands of the condensed matter context are taken into account.

A further kind of periodicity emerges, however, due to the consideration of a heat bath, as required by the KMS [7,8] conditions: the periodicity in the imaginary time. The simultaneous occurrence of both kinds of periodicities are considered here. In order to keep a close parallelism with the infinite space case we employ thermofielddynamics methods. These methods have been considered quite recently by us [9,10], in the bosonization of infinite space fields at finite temperature and have lead to a much transparent description, as has been witnessed in [11], which emphasizes the similarities with the zero temperature case. This work generalizes the thermofieldbosonization procedures to the case of concurrence of both kinds of periodicities and with this aim it differs from the contributions to the bosonization of compact space fields reported in [12].

The paper is organized as follows: we begin in Section 2 by considering the construction of free fields in compact spaces, with periodic boundary conditions. This streamlines the notation and makes the paper self contained. In Section 3 we describe the bosonization of the periodic fields at zero temperature. The comparison with the bosonization of fields in infinite space and finite temperature will be stressed. In Section 4 we add temperature and present compact expressions for the fields and their 2-point functions within thermofielddynamics. In Section 5 the bosonization is discussed in the case of finite temperature and a formalism to encode the contribution of the heat bath is developed. After discussing the results in Section 6 and presenting some technical details in the first two appendices, in Appendix C we discuss the completeness of the operator bosonization by computing the four point fermionic functions using the bosonization prescription.

2. Free fields with periodic conditions

We will adopt the strategy of building models of fields with periodic boundary by imposing the minimal requirements besides the ones borrowed from relativistic quantum field theory. Let us start by presenting the quantization of the Hermitian scalar field in two space–time dimensions with periodic boundary conditions.

In terms of light-cone variables, $x^\pm = x^0 \pm x^1$ (and $\partial_\pm = \partial_0 \pm \partial_1$) the scalar field obeys the equation of motion $\partial_+\partial_-\phi = 0$, so that $\phi(x) = \phi(x^+) + \phi(x^-)$. Imposing spatially periodic boundary conditions, $\phi(x^\pm) = \phi(x^\pm + L)$, the mode expansion turns out to be given by

$$\phi(x^\pm) = \sum_{k=1}^{\infty} \frac{1}{2\sqrt{\pi k}} \left[e^{-i\frac{2\pi k}{L}x^\pm} e^{-\epsilon k} a_k^\pm + e^{i\frac{2\pi k}{L}x^\pm} e^{-\epsilon k} a_k^{\pm\dagger} \right], \tag{2.1}$$

where we have inserted the ultraviolet convergence factor ϵ.

In order to obtain the equal-time canonical commutation relations, $\left[\phi(x), \dot\phi(x')\right]_{x^1\to x'^1} \approx i\delta(x^1 - x'^1)$, we require that $\left[a_k^\pm, a_{k'}^{\pm\dagger}\right] = \delta_{k',k}$. This results, however, in

$$\left[\phi(x^\pm), \dot\phi(x'^\pm)\right] = \frac{i}{2L} \lim_{\epsilon\to 0} \sum_{k=1}^{\infty} \left[e^{-\left(\frac{i2\pi\,\Delta x^\pm}{L}+2\epsilon\right)k} + e^{\left(\frac{i2\pi\,\Delta x^\pm}{L}-2\epsilon\right)k} \right] = \frac{i}{2}\delta_L(\Delta x^\pm) - \frac{i}{2L} \tag{2.2}$$

where δ_L represents the delta function with period L, $\delta_L(x+L) = \delta_L(x)$, and $\Delta x = x - x'$. The constant term of the last right side term, which spoils the strict canonical commutation relations, can be evaded by adding proper zero modes terms in Eq. (2.1).

From the mode expansion (2.1) we obtain for the two-point function

$$D(\Delta x^\pm) \equiv \langle 0|\phi(x^\pm)\phi(x'^\pm)|0\rangle = \sum_{k=1}^{\infty} \frac{1}{4\pi k} e^{-\left(\frac{i2\pi\,\Delta x^\pm}{L}+2\epsilon\right)k} = \frac{-1}{4\pi} \ln\left(1 - e^{-2\epsilon - \frac{i2\pi\,\Delta x^\pm}{L}}\right), \tag{2.3}$$