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Entropic uncertainty relation in de Sitter space

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HIGHLIGHTS

- The uncertainty principle could be reduced and quantified by a new Entropic Uncertainty Relation (EUR).
- By the open quantum system approach, we explore how the nature of the de Sitter (dS) space affects the EUR.
- For freely falls case, the entropic uncertainty acquires an increase resulting from the intrinsic thermal nature of the dS space.
- For static case, both the intrinsic thermal nature of the dS space and the Unruh effect brings effect on entropic uncertainty.
- The higher the temperature, the greater the uncertainty and the quicker the uncertainty reaches its maxima value.

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ABSTRACT

The uncertainty principle restricts our ability to simultaneously predict the measurement outcomes of two incompatible observables of a quantum particle. However, this uncertainty could be reduced and quantified by a new Entropic Uncertainty Relation (EUR). By the open quantum system approach, we explore how the nature of de Sitter space affects the EUR. When the quantum memory A freely falls in the de Sitter space, we demonstrate that the entropic uncertainty acquires an increase resulting from a thermal bath with the Gibbons–Hawking temperature. And for the static case, we find that the temperature coming from both the intrinsic thermal nature of the de Sitter space and the Unruh effect associated with the proper acceleration of A also brings effect on entropic uncertainty, and the higher the temperature, the greater the uncertainty and the quicker the uncertainty reaches the maximal value. And finally the possible mechanism behind this phenomenon is also explored.

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1. Introduction

Heisenberg's uncertainty relation [1], which lies at the heart of understanding quantum mechanics, provides a dramatic illustration of a qualitative distinction between quantum and classical physics. This principle states that there is general irreducible lower bound on the uncertainty in the result of simultaneous measurement of two conjugate quantum mechanical variables, such as position and momentum, and more precisely, the product of the uncertainties in such two measurements is at least of order $\hbar$, or equivalently, there is an upper bound on the accuracy with which the values of noncommunity observables can be simultaneously prepared.

Due to the appearance of information theory, a more natural choice to measure uncertainty is based on entropy [2–6]. For non-commuting observables Q and R, Deutsch [3] has described the relation as

$$S_H(Q) + S_H(R) \geq -2\log_2 \frac{1}{2}(1+c), \tag{1.1}$$

where $S_H = -\sum_j p(j)\log_2 p(j)$, Q and R denote two Hermitian operators representing physical observables in an N-dimensional Hilbert space with $\{|a_j\rangle\}$ and $\{|b_j\rangle\}$ $(j = 1, \dots, N)$ the respective complete sets of normalized eigenvectors and $c = \max_{i,j} |\langle a_i|b_j\rangle|$. Particularly, Kraus [4] suggested that this relation may be improved to

$$S_H(Q) + S_H(R) \geq \log_2 \frac{1}{c^2}. \tag{1.2}$$

A distinct advantage of these relations, (1.1) and (1.2), over the standard deviations is that the right-hand side is independent of the state of the system when the two measurements Q and R do not share any common eigenvector, i.e., it gives a fixed lower bound. So, they provide us a more general framework to quantify uncertainty.

However, using previously determined quantum information about the measured system, the above uncertainty bound could be violated. To overcome this defect, recently Refs. [7,8] have given a stronger Entropic Uncertainty Relation (EUR) based on conditional entropy theoretically. Furthermore, several experiments [9,10] have been performed to confirm this EUR. For an entangled quantum system consisting of interesting particle B and its quantum memory A, which is a device that might be available in the not-too-distant future and could store the information of the entanglement between particles [11], the conditional entropy EUR is shown as

$$S_v(Q|A) + S_v(R|A) \geqslant \log_2 \frac{1}{c^2} + S_v(B|A), \tag{1.3}$$

where $S_v(B|A) = S_v(\rho_{AB}) - S_v(\rho_A)$ is the conditional von Neumann entropy. In the extreme case, i.e., A and B are maximally entangled, it is able to predict the outcomes precisely. On the other hand, if A and B are not entangled, the bound in (1.2) is recovered. The generalization of the EUR (1.3) to Rényi entropy has also been given [12,13]. Other studies from various views can be found in [14–16].

It is well known that every quantum system, whatever it is, in a realistic regime is inevitably in contact with environments. As a result, the considered quantum system has to suffer a decoherence or dissipation. So the nature of environment plays a key role in dominating the evolution of the quantum system, as well as the quantum-memory-assisted EUR [17]. Besides the generally studied noisy channels, such as bit flip, noises resulting from the motions of observers or gravitational field are also a very important branch of quantum noisy channels. Especially, such noises directly relate to the nature of spacetime, such as Hawing effect, and allow us to incorporate the concepts of quantum information into relativistic settings. This combination has recently resulted in an entire novel field of physics, relativistic quantum information [18]. Its aim is to answer questions about the overlap of relativity and the manipulation of information stored in quantum system, provide us a more complete frame to understand quantum information theoretically, and more importantly be a guidance for future realistic quantum information assignments in curved spacetime. So such works are very meaningful. J. Feng et al. in recent work [19] have studied how the Unruh effect affects the EUR, which is the first try to discuss how the motion of the observer affects the Heisenberg's limit. However, their analysis is confined in the flat spacetime and the effects result only from the motion